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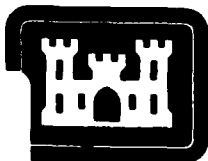
ARMY ENGINEER WATERWAYS EXPERIMENT STATION VICKSBURG--ETC F/G 8/13  
A ONE-DIMENSIONAL PLANE WAVE PROPAGATION CODE FOR LAYERED RATE---ETC(U)  
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# A ONE-DIMENSIONAL PLANE WAVE PROPAGATION CODE FOR LAYERED RATE-DEPENDENT HYSTERETIC MATERIALS

by

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September 1981

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Recent experimental data on soils showing that the application of loads with submillisecond rise times results in significant rate-dependent com- pressibility behavior have prompted the need for a one-dimensional plane wave propagation code for layered rate-dependent hysteretic or visco-compacting materials. Accordingly, an explicit one-dimensional finite element code named ONED3P has been developed which incorporates a three-parameter (spring (Continued)		

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ABSTRACT (Continued).

and dashpot) mechanical model consisting of a linear spring and dashpot in series coupled in parallel with a piecewise linear hysteretic or compacting spring.

ONED3P allows for the analysis of a column of multilayered visco-compacting soils loaded by a digitized surface pressure-time history. Any set of consistent units may be used with this code and results may be obtained in the form of stress-, strain-, acceleration-, velocity-, and/or displacement-time histories as well as stress-strain curves using standard Calcomp software.

Several demonstration problems were calculated using ONED3P to evaluate its features and capabilities against (a) available analytical solutions for viscous and nonviscous problems, (b) other code solutions, and (c) measurements from field experiments that evince rate-dependent soil behavior. In general, results were extremely good.

Special attention was given to the effects of loading rate or frequency on wave speeds in viscous media and to methods of deriving the ONED3P model parameters from laboratory material property test data. Program listings and a user's guide for ONED3P are included in Appendix A.

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## PREFACE

The investigation reported herein was sponsored by the Assistant Secretary of the Army (R&D) under Department of the Army Project 4A161101A91D, In-House Laboratory Independent Research Program.

The study was conducted by Mr. J. O. Curtis of the Structures Laboratory (SL), U. S. Army Engineer Waterways Experiment Station (WES), during the period November 1979-September 1981, under the general direction of Dr. J. G. Jackson, Jr., Chief of the Geomechanics Division (SD). Technical guidance was provided by Dr. Behzad Rohani, SD. The report was prepared by Mr. Curtis.

Special acknowledgement is given to Drs. Rohani and J. S. Zelasko for their technical review of this report and to Mr. J. D. Cargile for conducting many of the computer calculations.

Director of WES during this investigation was COL Nelson P. Conover, CE. Technical Director was Mr. F. R. Brown. Messrs. Bryant Mather and W. J. Flathau were Chief and Assistant Chief, respectively, of the Structures Laboratory.

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A ONE-DIMENSIONAL PLANE WAVE PROPAGATION  
CODE FOR LAYERED RATE-DEPENDENT  
HYSTERETIC MATERIALS

PART I: INTRODUCTION

Background

1. Material property test data recently acquired in the Geomechanics Division of the Structures Laboratory at the U. S. Army Engineer Waterways Experiment Station (WES) have revealed that the dynamic compressibility response of various soils subjected to loadings with submillisecond rise times is both qualitatively and quantitatively different than their response to slower loadings.<sup>1</sup> Reference 1 cites several examples of test results in which the stress-strain response of soils was much stiffer during rapid loading conditions than for quasi-static experiments.

2. Furthermore, field measurements recently acquired during shallow-buried structures experiments (References 2 and 3) indicated that for surface loading rise times on the order of 0.01 to 0.1 msec, high-amplitude (10 to 40 MPa) stress waves traveled faster through the sand cover above the structures than would be predicted from seismic velocity data or from uniaxial strain test data generated in the laboratory using loadings with rise times on the order of a few milliseconds.

Purpose and Scope

3. The purpose of this report is to describe the development and evaluation of a one-dimensional plane stress wave propagation code called ONED3P which treats layered, nonlinear, rate-dependent, hysteretic materials.

4. A description of the constitutive relationship used in ONED3P, which is represented by a three-parameter mechanical model, is given in

Part II. Part III contains a description of the major features of ONED3P, including its solution algorithm and its treatment of different boundary conditions. The capabilities of the code are checked against available analytical solutions and other code calculations in Part IV. Part V describes how the model parameters can be evaluated from laboratory and field data and goes on to compare ONED3P calculation results with those from field experiments. Finally, a user's guide is presented in Appendix A.



## PART II: MODEL DESCRIPTION

### Mechanical Model

5. In general, a rate-dependent constitutive model should relate stress, strain, stress rate, and strain rate in the following functional form:

$$\dot{\sigma} = f(\sigma, \epsilon, \dot{\epsilon}) \quad (1)$$

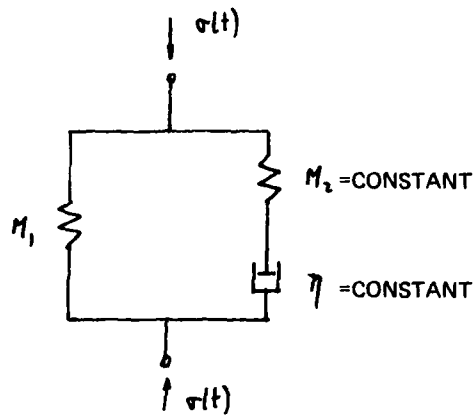
where the dot indicates time differentiation.

6. The literature (References 4 and 5) shows that numerous relationships among forces, displacements, loading rates, and velocities may be written by developing the governing equations for various combinations of linear mechanical elements; namely, springs and dashpots. The springs generate forces (or stresses) proportional to displacements (or strains), while the dashpots generate forces proportional to velocity (or strain rate). An example of a mechanical model whose governing equation looks like Equation 1 but which is still relatively simple to work with is shown in Figure 1a. Furthermore, Reference 1 has already demonstrated that such a model can be used to simulate the rate-dependent stress-strain response of a single particle.

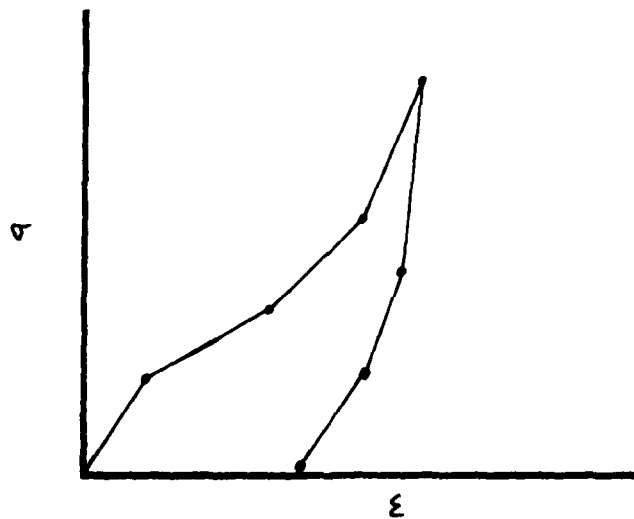
7. It can be shown that the equation which governs the behavior of this three-parameter model is<sup>4</sup>

$$\dot{\sigma} + \frac{M_2}{\eta} \sigma = \frac{M_2 M_1}{\eta} \epsilon + (M_2 + M_1) \dot{\epsilon} \quad (2)$$

The  $M_1$  and  $M_2$  functions in Equation 1 bound the stress-strain behavior of the material in the following ways. First, for very slow loading rates ( $\dot{\sigma}, \dot{\epsilon} \rightarrow 0$ ),  $M_1$  describes the complete stress-strain response and, hence, represents the "static" behavior of the material. On the other hand, for extremely fast loading rates, the dashpot acts as though it were rigid and the sum of  $M_1$  and  $M_2$  governs the model behavior.



a. ARRANGEMENT OF SPRINGS AND DASHPOT  
IN MECHANICAL MODEL



b. STATIC STRESS-STRAIN CURVE DEFINING THE  $M_1$  FUNCTION

Figure 1. Details of the proposed visco-compacting mechanical model for soils

Therefore  $M_1 + M_2$  represents the upper bound of material stiffness. Any material response between these bounds is then controlled by the value assigned to  $\eta$ .

#### Finite Difference Form of the Governing Equation

8. Equation 1 is written for constant material property parameters  $M_1$ ,  $M_2$ , and  $\eta$  and deals with total values of stress and strain and their time derivatives. To accommodate nonlinear properties, Equation 2 was written in incremental form as

$$\Delta\sigma + \frac{\eta}{M_2} \Delta\dot{\sigma} = M_1 \Delta\epsilon + \eta \left( 1 + \frac{M_1}{M_2} \right) \Delta\dot{\epsilon} \quad (3)$$

wherein  $M_1$ ,  $M_2$ , and  $\eta$  are considered to be constants within each increment of time. In fact, as a first-order approximation to obtaining a model of the visco-compacting material described in Part I,  $M_2$  and  $\eta$  are treated as constants for all time.  $M_1$  is described by a piecewise linear stress-strain curve (with an arbitrary number of segments) which has separate loading and unloading behavior (Figure 1b).

9. Using the subscript  $i+1$  to refer to a new point in time,  $i$  to refer to the present time, and  $i-1$  to refer to the previous point in time, the incremental terms in Equation 3 may be written:

$$\left. \begin{aligned} \Delta\sigma &= \sigma_{i+1} - \sigma_i \\ \Delta\epsilon &= \epsilon_{i+1} - \epsilon_i \\ \Delta\dot{\sigma} &= \sigma_{i+1/2} - \sigma_{i-1/2} = \frac{\sigma_{i+1} - \sigma_i}{\Delta t_n} - \frac{\sigma_i - \sigma_{i-1}}{\Delta t_o} \\ \Delta\dot{\epsilon} &= \frac{\epsilon_{i+1} - \epsilon_i}{\Delta t_n} - \frac{\epsilon_i - \epsilon_{i-1}}{\Delta t_o} \end{aligned} \right\} \quad (4)$$

where  $\Delta t_n = t_{i+1} - t_i$  and  $\Delta t_o = t_i - t_{i-1}$ . Substituting Equation 4 into Equation 3 yields the final incremental form of the

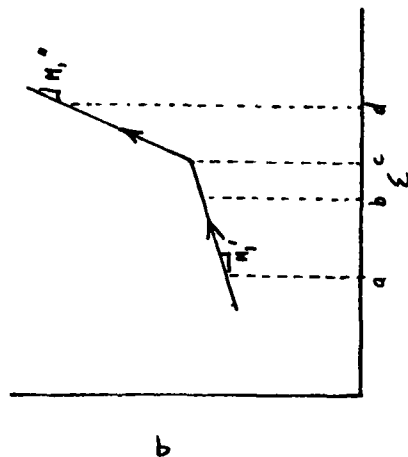
constitutive relationship used in ONED3P:

$$\sigma_{i+1} = \sigma_i \left( 1 + \frac{\frac{\eta}{M_2 \Delta t_o}}{1 + \frac{\eta}{M_2 \Delta t_n}} \right) - \sigma_{i-1} \left( \frac{\frac{\eta}{M_2 \Delta t_o}}{1 + \frac{\eta}{M_2 \Delta t_n}} \right) + (\epsilon_{i+1} - \epsilon_i) \times \left[ \frac{M_1 + \frac{\eta}{\Delta t_n} \left( 1 + \frac{M_1}{M_2} \right)}{1 + \frac{\eta}{M_2 \Delta t_n}} \right] - (\epsilon_i - \epsilon_{i-1}) \left[ \frac{\frac{\eta}{\Delta t_o} \left( 1 + \frac{M_1}{M_2} \right)}{1 + \frac{\eta}{M_2 \Delta t_n}} \right] \quad (5)$$

10. As long as  $M_1$  remains constant over any two consecutive time increments,  $\Delta t_o$  is equal to  $\Delta t_n$ . If, however,  $M_1$  changes within a time increment the following technique is used to evaluate the new state of stress. Consider the diagrams shown in Figure 2 where the strain of some piece of the material has proceeded from point a to point b to point d on two consecutive time increments. There is a change in  $M_1$  between points b and d. ONED3P automatically breaks the time increment from b to d into two (or more, if necessary) subintervals which are proportional to the strain segments  $(\epsilon_c - \epsilon_b)$  and  $(\epsilon_d - \epsilon_c)$ . The code then proceeds as shown in Figure 2 to calculate the new stress state in two steps (or more).

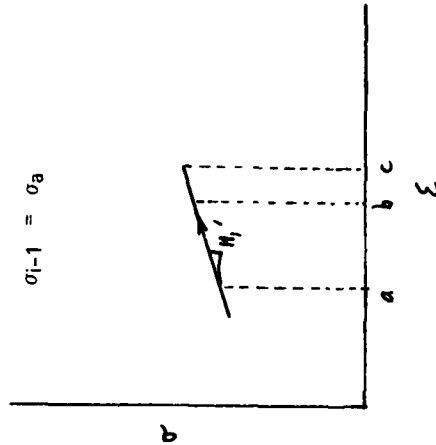
11. Other techniques for handling a change in the  $M_1$  function were tried including (a) rewriting Equation 3 to include variable  $M_1$  values, (b) choosing one  $M_1$  value or the other to be used in Equation 5, and (c) computing an average value of  $M_1$  over an interval. Assuming that there were no programming errors, none of the other techniques gave results as consistently stable as did the method shown in Figure 2.

$$\begin{aligned}\Delta t_0 &= t_b - t_a \\ \Delta t_n &= t_d - t_b \\ \epsilon_{i+1} &= \epsilon_d \\ \epsilon_i &= \epsilon_b \\ \epsilon_{i-1} &= \epsilon_a \\ \sigma_i &= \sigma_b \\ \sigma_{i-1} &= \sigma_a\end{aligned}$$



==

$$\begin{aligned}\Delta t_0 &= t_b - t_a \\ \Delta t_n &= t_c - t_b \\ M_1 &= M_1' \\ \epsilon_{i+1} &= \epsilon_c \\ \epsilon_i &= \epsilon_b \\ \epsilon_{i-1} &= \epsilon_a \\ \sigma_i &= \sigma_b \\ \sigma_{i-1} &= \sigma_a\end{aligned}$$



+

$$\begin{aligned}\Delta t_n &= t_d - t_c \\ M_1 &= M_1'' \\ \epsilon_{i+1} &= \epsilon_d \\ \epsilon_i &= \epsilon_{i-1} = \epsilon_c \\ \sigma_i &= \sigma_{i-1} = \sigma_c\end{aligned}$$

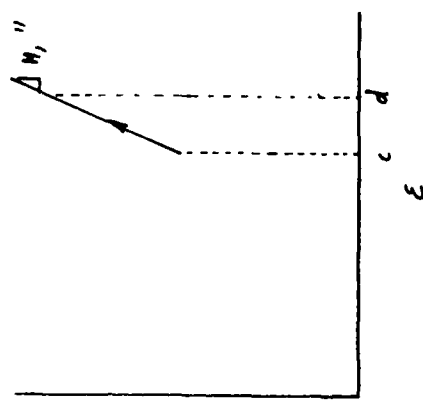


Figure 2. Handling a change in  $M_1$  during a given time increment

### PART III: CODE DESCRIPTION

#### Solution Algorithm

12. Spatially, ONED3P employs two-node isoparametric one-dimensional finite elements in which displacement is assumed to vary linearly between the nodes. This assumption leads to a constant strain and, hence, constant stress within each element. Furthermore, half of the elements's mass is assigned to each node and because each element is assumed to have a unit cross-sectional area, the stress in each element may be replaced by node point loads (or forces) equal in magnitude to the element stress. Nodal accelerations are found by summing the forces acting on each node and dividing by the node mass.

13. This scheme allows for a simple visual interpretation of what the ONED3P code deals with. Figure 3a shows a column of continuous material, the horizontal dimensions of which have no meaning (since the code works with a unit cross-sectional area). Figure 3b shows the equivalent system of lumped masses and mechanical elements with which ONED3P actually works.

14. The solution algorithm for the equivalent system is shown graphically in Figure 4. New nodal velocities (V) and displacements (d) are calculated by a simple linear integration scheme:

$$\left. \begin{aligned} V_{\text{new}} &= V_{\text{old}} + a_{\text{new}} \cdot \Delta t \\ d_{\text{new}} &= d_{\text{old}} + V_{\text{new}} \cdot \Delta t \end{aligned} \right\} \quad (6)$$

where "a" stands for acceleration. A more exact integration scheme was tried but led to numerical instabilities.\*

---

\* It is the author's belief that the linear integration scheme used in ONED3P serves to make the normally lagging response of an element (due to the finite spacing of nodes and the use of a finite time increment) "catch up" to the true solution by computing larger velocity increments while accelerating and smaller increments when decelerating.

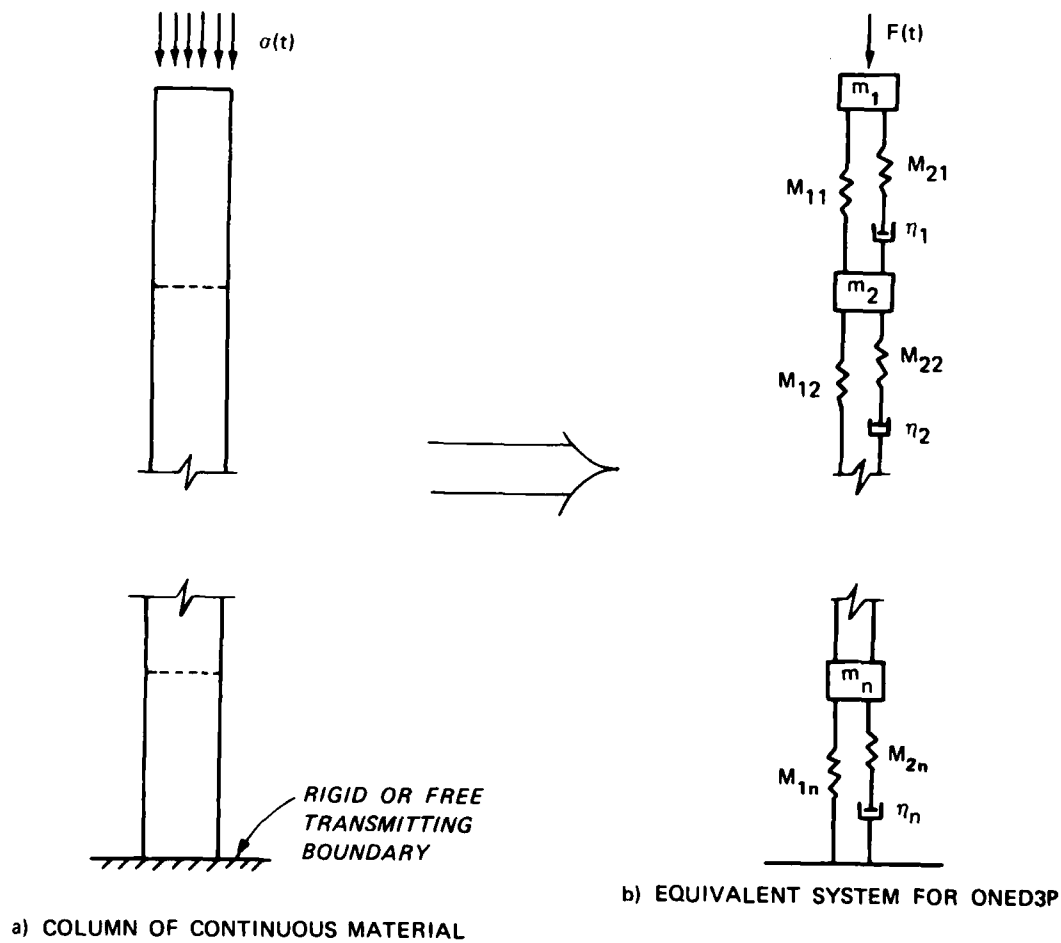


Figure 3. ONED3P interpretation of a typical one-dimensional problem

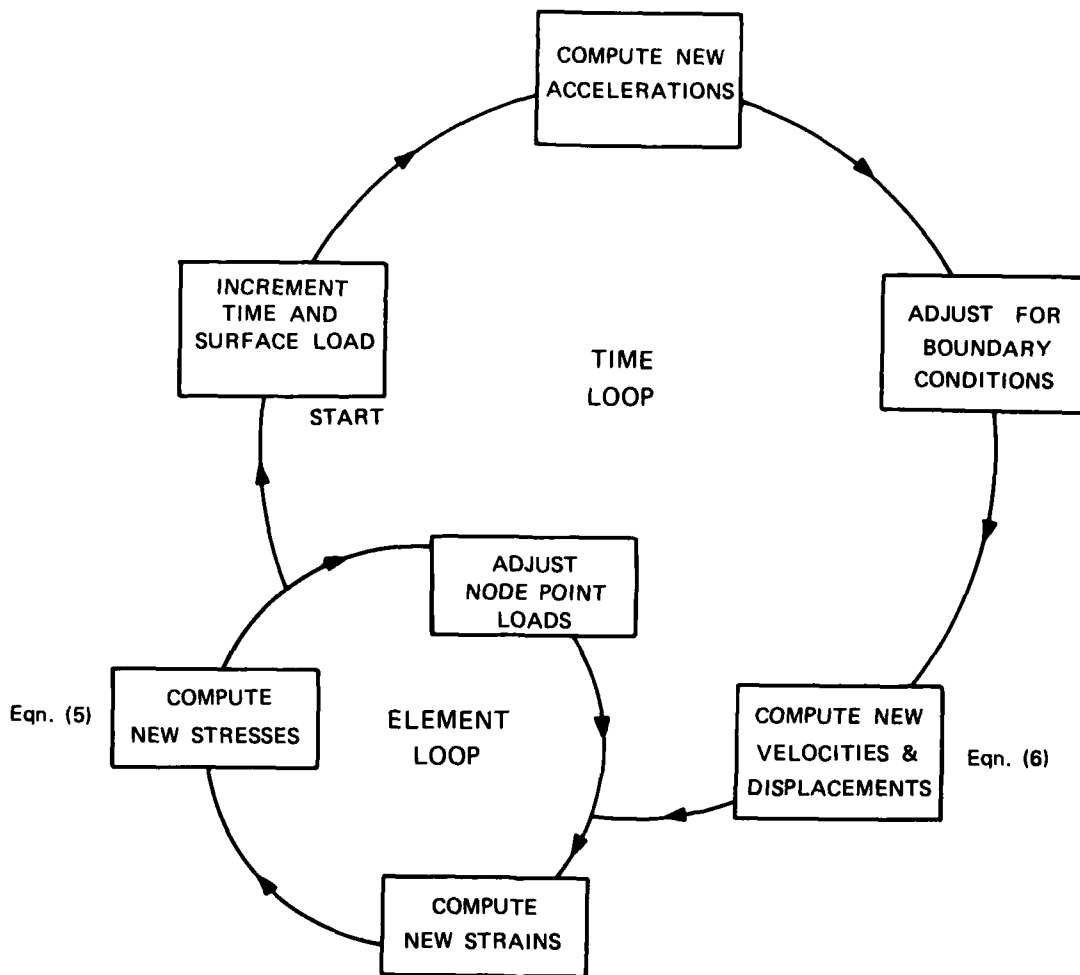


Figure 4. Solution algorithm for ONED3P

#### Boundary Conditions

15. Fixed and free bottom boundary conditions are handled in ONED3P just as they are in all finite element codes; namely, the acceleration for a fixed-surface node is always set equal to zero while a free-surface node is treated like any other node within the material column.

16. An approximate transmitting bottom boundary has also been incorporated into ONED3P. The expression for particle velocity and strain from linear elastic one-dimensional wave propagation theory is:<sup>5</sup>



$$V = C\epsilon \quad (7)$$

where  $C$  is the wave speed which, under uniaxial strain conditions, is

$$C = \sqrt{\frac{M}{\rho}} \quad (8)$$

$M$  being the constrained modulus and  $\rho$  the density of the material. To implement the transmitting boundary, Equation 7 was written incrementally as:

$$V_{\text{new}} = V_{\text{old}} + C(\epsilon_{\text{new}} - \epsilon_{\text{old}}) \quad (9)$$

where the strain at the boundary node was taken to be the current strain in the last element.

17. The wave speed is computed anew at each time step as a function of the current value of  $M_1$  in the last element using the following reasoning. First, the tangent modulus at any point on the dynamic stress-strain curve cannot be used because it can have negative values. Second, the current value of  $M_1$  was found to be insufficient for viscous problems in which the dynamic stress-strain curve was much different than the static curve. As expected, using  $M_1$  alone resulted in a soft boundary for highly viscous calculations. Obviously what is needed is a modulus which approaches  $M_1$  under nonviscous conditions but which takes into account the stiffer dynamic stress-strain response of highly viscous calculations. One measure of the dynamic response in a material is the net amount of energy absorbed at a given point--in other words, the area under the stress-strain curve. Based upon these observations, the value of  $M$  in Equation 8 was finally taken to be:

$$M = M_1 \frac{A_d}{A_s} \quad (10)$$

where

$A_d$  = the area under the loading portion of the dynamic stress-strain curve

$A_s$  = the area under the loading portion of the static curve  
This treatment gives stable results and has worked quite well under most conditions.

#### Surface Loading

18. Since the one-dimensional column being simulated by ONED3P is assumed to have a unit cross-sectional area, force and stress are synonymous and therefore a stress-time history may be applied directly to the surface node as a force-time history. To allow for generality of input, the surface forcing function used in ONED3P must be digitized and read by the code from a data file. With only a minor modification to the code, a velocity-time history could be applied to the surface as well.

#### Consistent Units

19. One convenient feature of ONED3P is that any set of consistent units may be used in the code. Consistent units are sets of units which do not require conversion factors to make calculations balance in a unit sense. Any of the sets of units shown in Table 1 may be used in ONED3P to generate equivalent results. Note that set D represents the set of units normally referred to as SI units.

#### Plotting of Results

20. ONED3P has been written to generate several types of plots using standard Calcomp software. Time histories of stress, strain, acceleration, velocity, and displacement may be obtained as well as plots of total dynamic stress versus strain. Further information on plots is available in Appendix A.

Table 1

## Consistent Sets of Units

Dimension	Sets of Units						
	A	B	C	D	E	F	G
Mass	g	g	g	kg	kg	kg	lb <sub>f</sub> -sec <sup>2</sup> /in.
Length	cm	cm	cm	m	m	km	in.
Time	sec	msec	μsec	sec	msec	sec	sec
Density	g/cm <sup>3</sup>	g/cm <sup>3</sup>	g/cm <sup>3</sup>	kg/m <sup>3</sup>	kg/m <sup>3</sup>	kg/km <sup>3</sup>	lb <sub>f</sub> -sec <sup>2</sup> /in. <sup>4</sup>
Velocity	cm/sec	cm/msec	cm/μsec	m/sec	m/msec	km/sec	in./sec
Force	dyne	Mdyne	10 <sup>+12</sup> dyne	newton	Mnewton	knewton	lb <sub>f</sub>
Stress	μbar	bar	Mbar	10 <sup>-5</sup> bar (1 pascal)	10 bar (1 Megapascal)	10 <sup>-2</sup> bar	lb <sub>f</sub> /in. <sup>2</sup>
Energy*	erg ≡ 10 <sup>-7</sup> joules	Merg ≡ 10 <sup>-1</sup> joules	10 <sup>12</sup> erg ≡ eu	joule ≡ watt-sec	Mjoule ≡ Mwatt-sec	kjoule ≡ kwatt-sec	lb <sub>f</sub> -in.

\* Energy is not dealt with in ONED3P, but was added to the table to make it "complete."

#### PART IV: ONED3P COMPARISONS WITH AVAILABLE SOLUTIONS

21. A variety of ONED3P calculations were performed on the WES G-635 and DPS/1 computer systems to demonstrate and evaluate how well the code presently functions. These demonstrations are presented herein.

22. Time increments and loading function rise times which led to stable calculations were selected using commonly accepted criteria;<sup>6</sup> namely,

$$\Delta t < \frac{\Delta z_{\min}}{C_{\max}} \quad (11)$$

$$t_r > \pi \frac{\Delta z_{\max}}{C_{\min}} \quad (12)$$

where  $\Delta z$  is the element size and the minimum and maximum wave speeds for each problem are determined by the smallest  $M_1$  value and the sum of  $M_2$  and the largest  $M_1$  value, respectively. Although loading functions which do not satisfy Equation 12 may be used, it was discovered that such calculations generated larger-than-expected wave speeds for elastic problems.

#### Nonviscous Problems

##### Linear elastic single-layered column

23. Figure 5 describes the problem geometry, material properties, surface loading conditions, and boundary conditions for four ONED3P test calculations that specify linear elastic material behavior. The first three calculations are for a single layer of material. Problems 1 and 2 were computed to test free and fixed bottom boundary conditions, respectively, while Problem 3 was run to determine whether or not a simple calculation could be made with a step load having no rise time.

24. Stress-, velocity-, and displacement-time histories for each

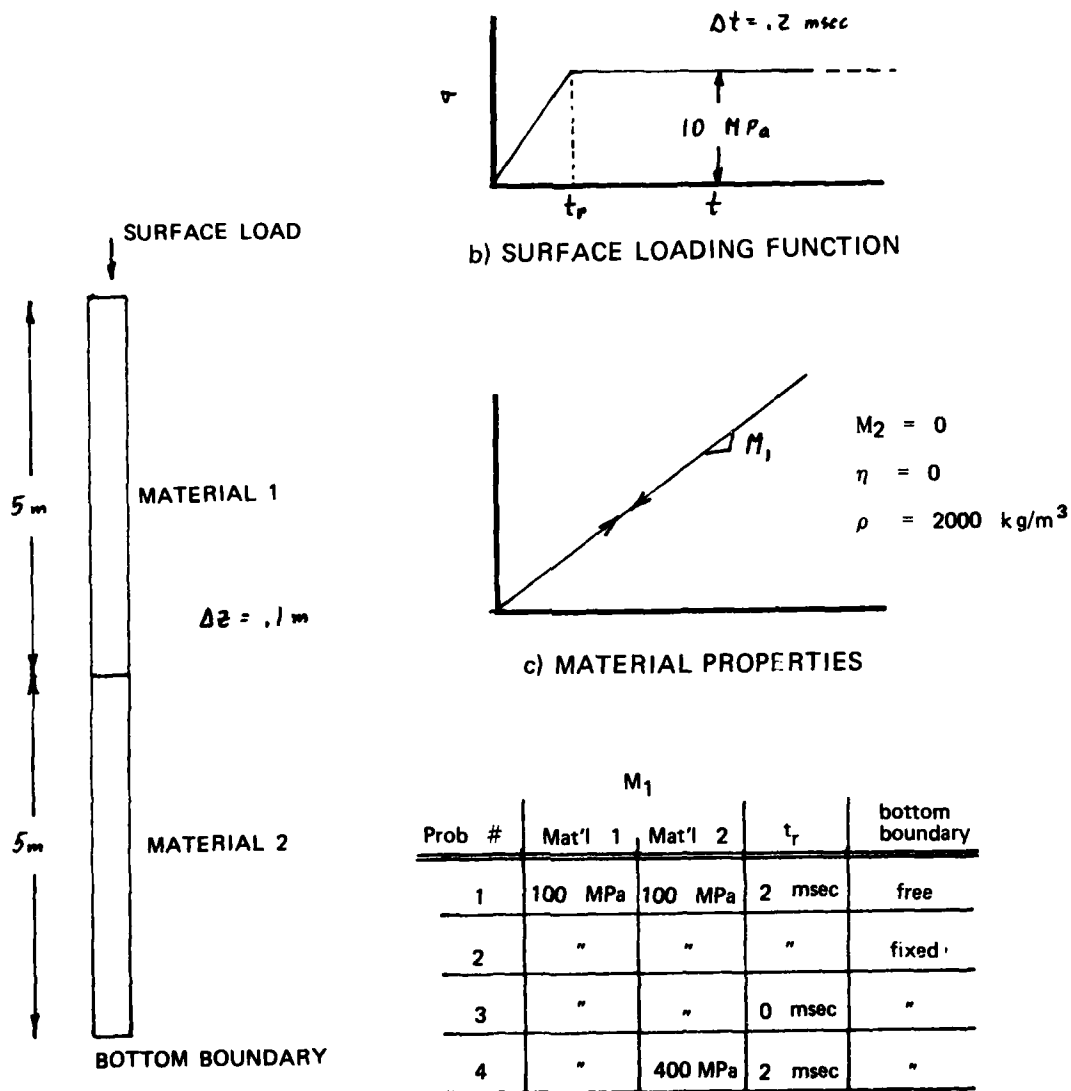


Figure 5. Problem descriptions for linear elastic calculations

of these three problems are contained in Figures 6 through 14. Exact solutions are superimposed on selected stress-time histories and clearly show that ONED3P can handle linear elastic calculations very well. Eliminating the loading function rise time in Problem 3 resulted in slightly greater stress and velocity oscillations than those in Problem 2 as well as greater wave speeds than expected, but the calculation still remained stable. This is not to say that all ONED3P calculations would be stable without a loading function rise time. Rather, the user is advised to use the rise time stability criterion (Equation 12) for all computations.

Linear elastic multilayered column

25. Figure 5 also contains a description of Problem 4, which is a column of two material layers, the bottom layer having twice the impedance of the top layer. Results for this calculation are shown in Figures 15 through 17 and, once again, the code does an excellent job of matching the analytical solution.

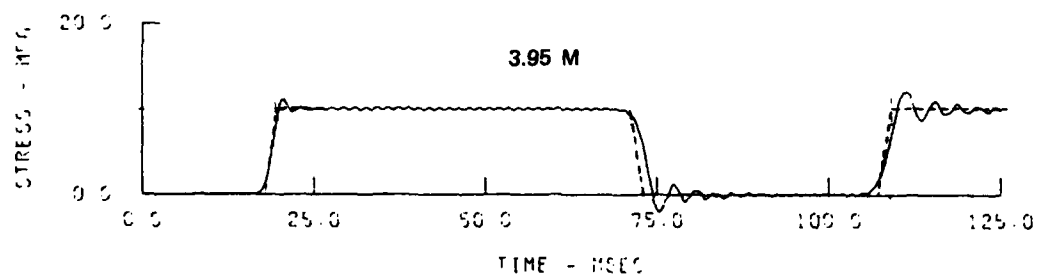
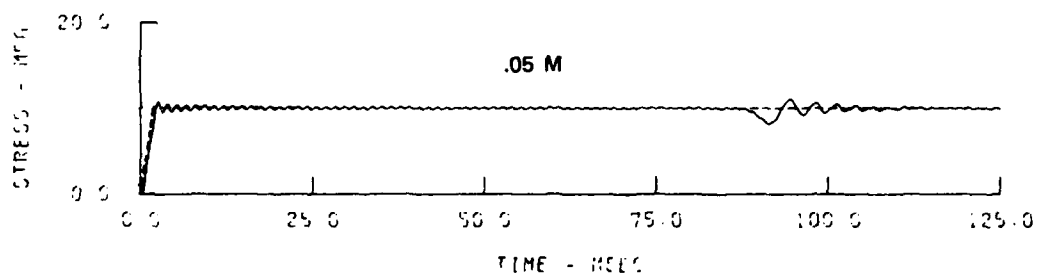
Linear hysteretic  
single-layered column

26. Salvadori et al.<sup>(7)</sup> developed an analytical solution for one-dimensional stress wave propagation through linear hysteretic material which was applied to Problem 5 described in Figure 18. Since the analytical solution was for a semi-infinite medium, Problem 5 presented an opportunity to test the transmitting boundary in ONED3P on something other than a linear elastic material.

27. Calculation results are compared with analytical results in Figures 19 through 21. Agreement is excellent. Note that in the stress-time histories only a slight bump occurs in the response of each element at the times when waves reflected off the boundaries would normally pass through the element.

Nonlinear hysteretic  
single-layered column

28. The only available solution for stress wave propagation through highly nonlinear hysteretic material was one generated by the ONED code and presented in Reference 8. This problem, designated



**LEGEND**  
 — ONED3P RESULTS  
 - - - EXACT SOLUTION

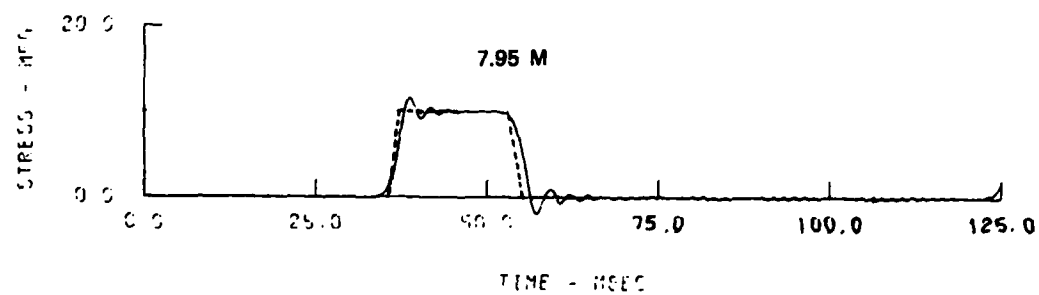


Figure 6. Stress-time histories for Problem 1

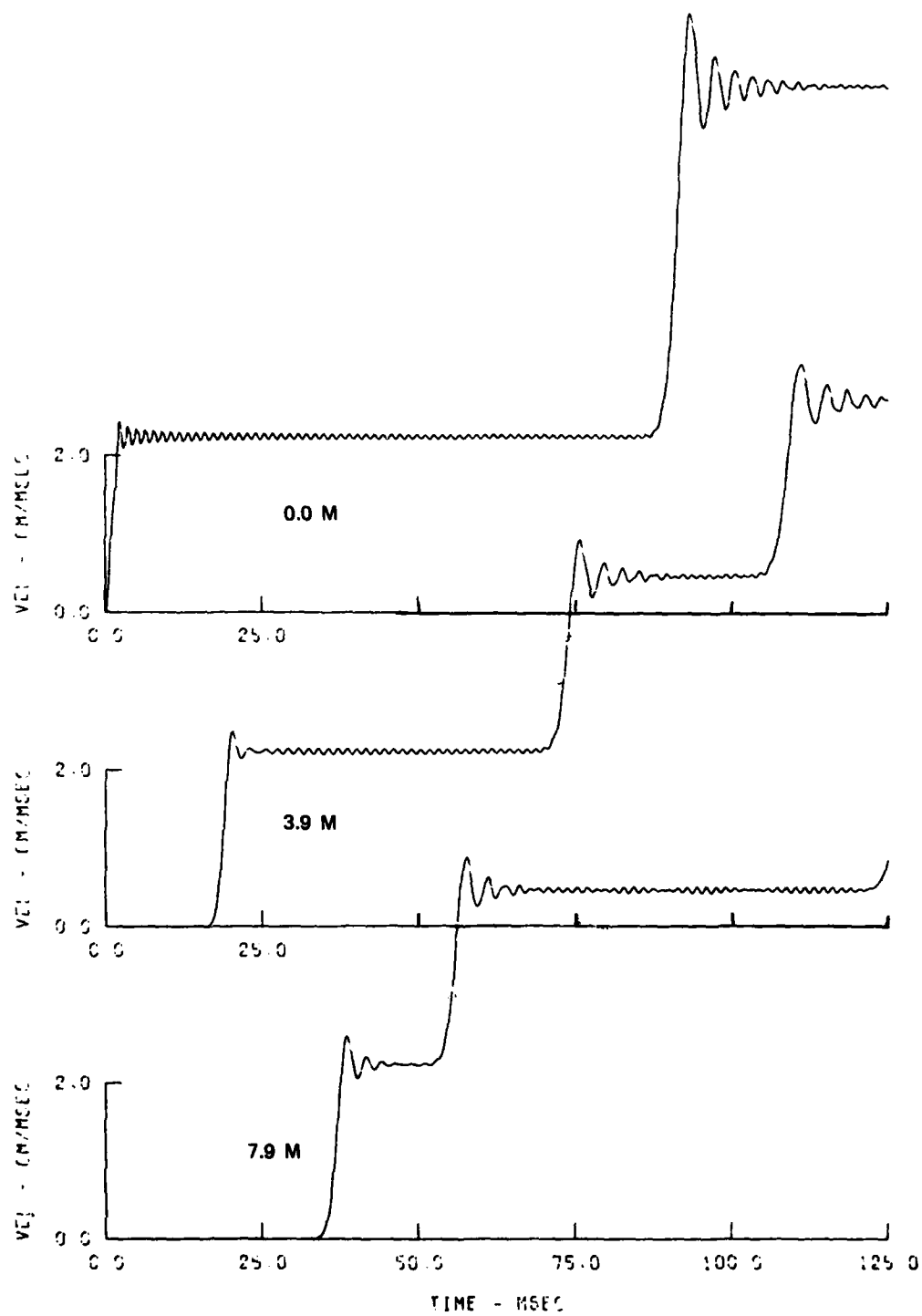


Figure 7. Velocity-time histories for Problem 1



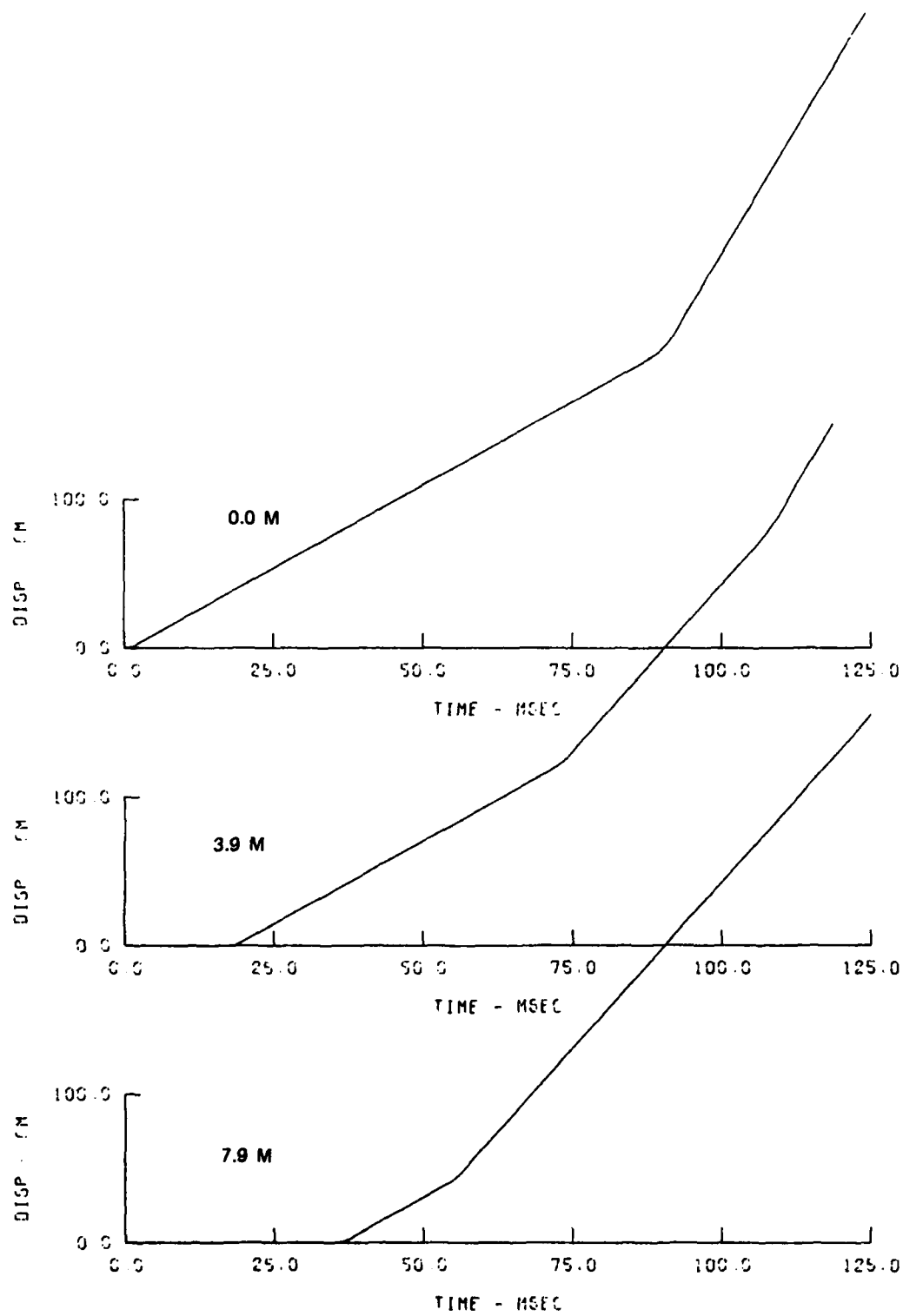


Figure 8. Displacement-time histories for Problem 1

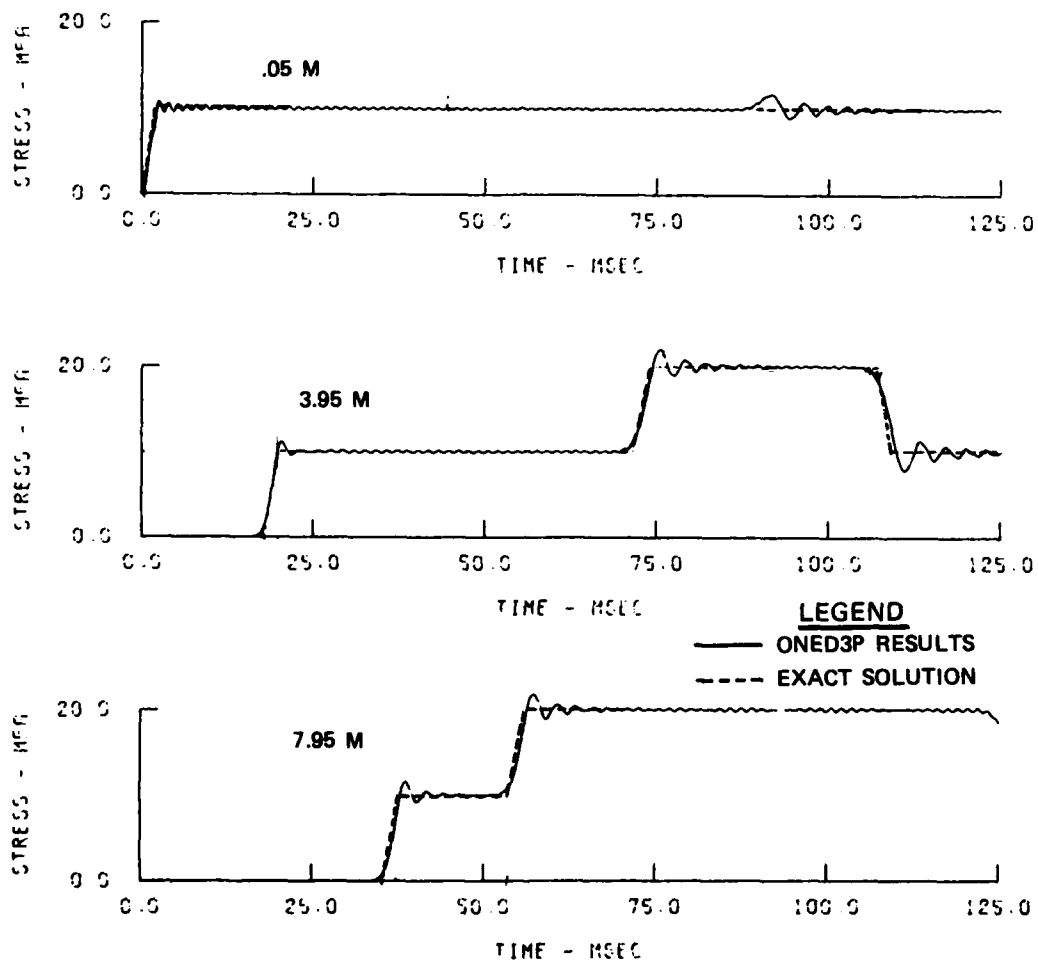


Figure 9. Stress-time histories for Problem 2

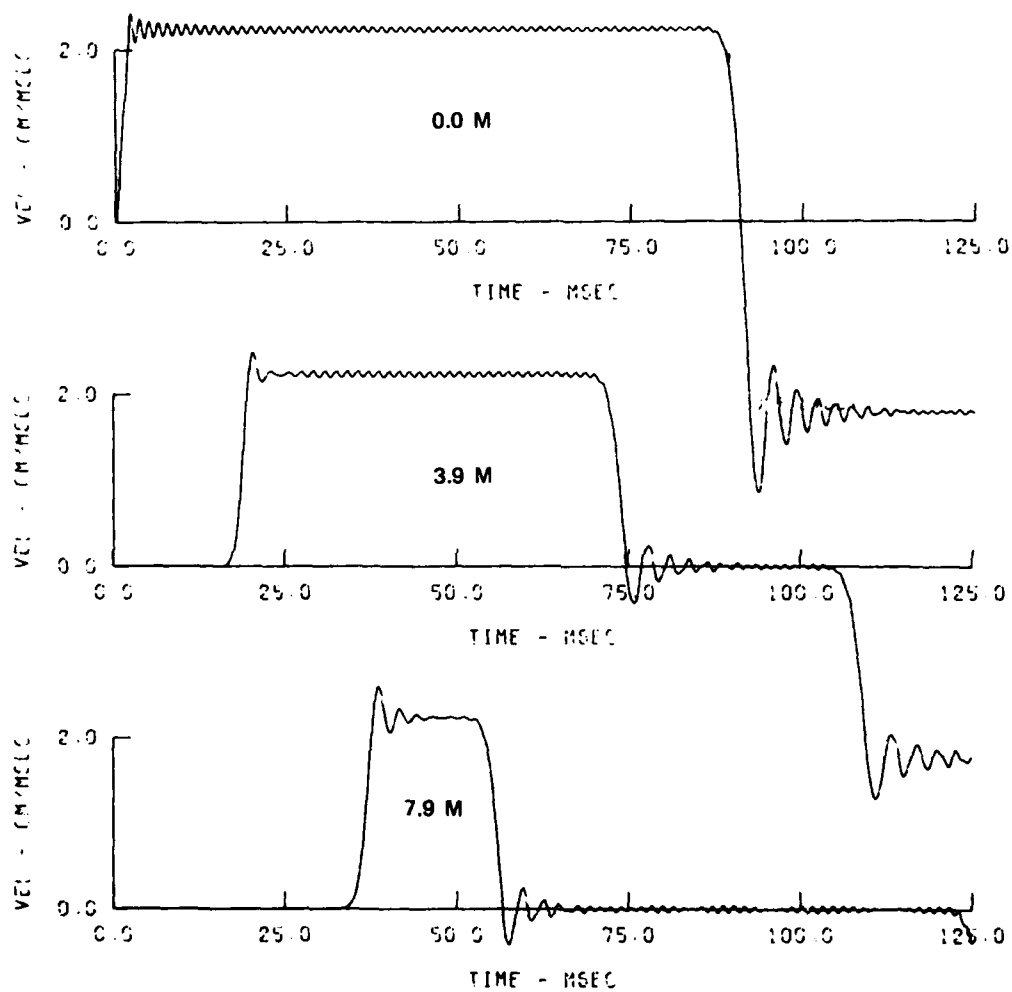


Figure 10. Velocity-time histories for Problem 2

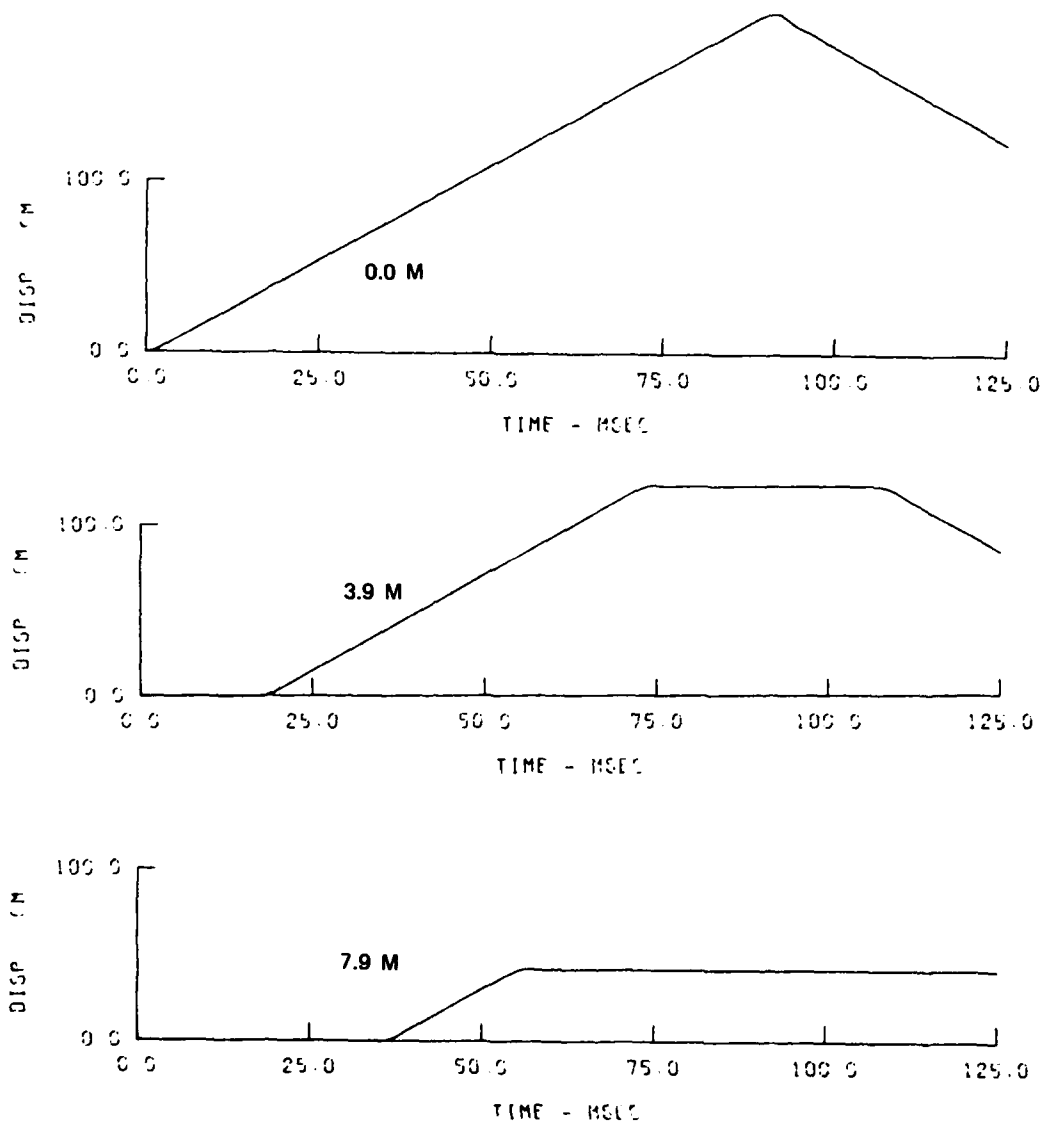


Figure 11. Displacement-time histories for Problem 2

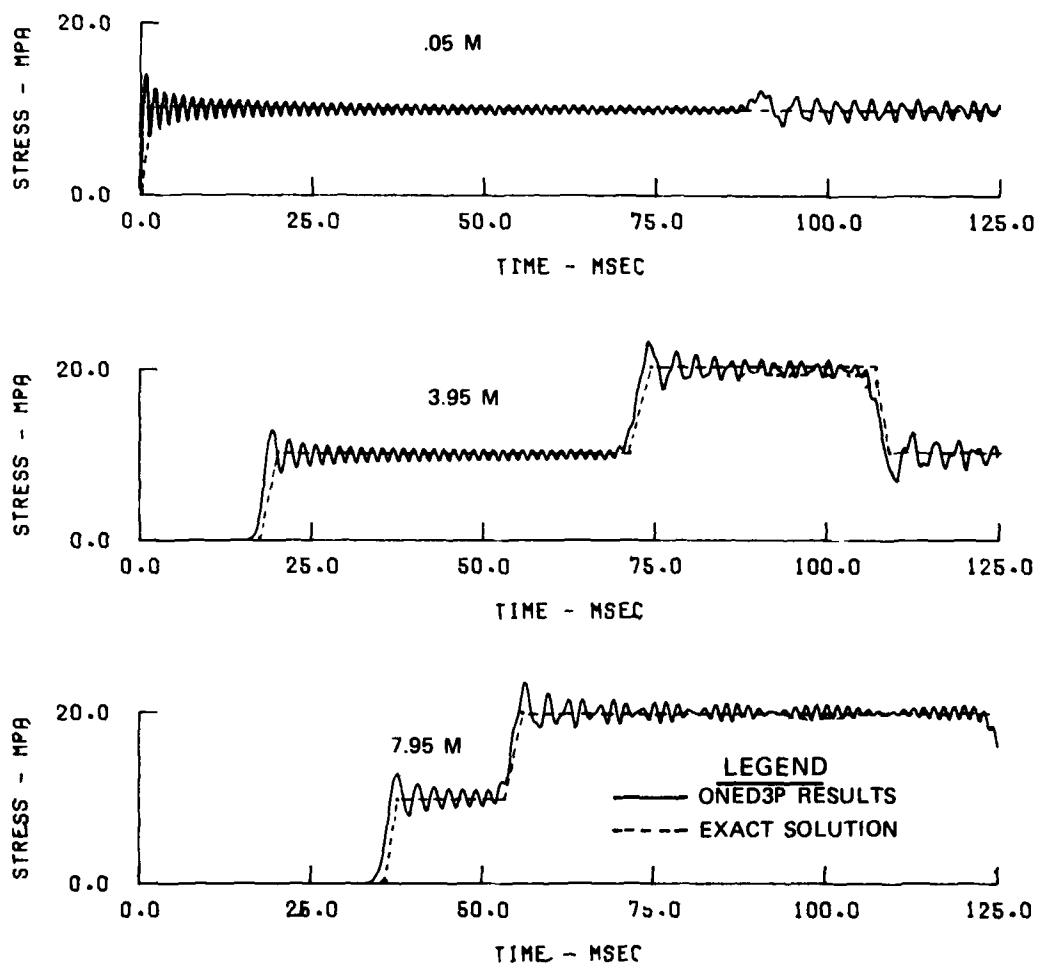


Figure 12. Stress-time histories for Problem 3

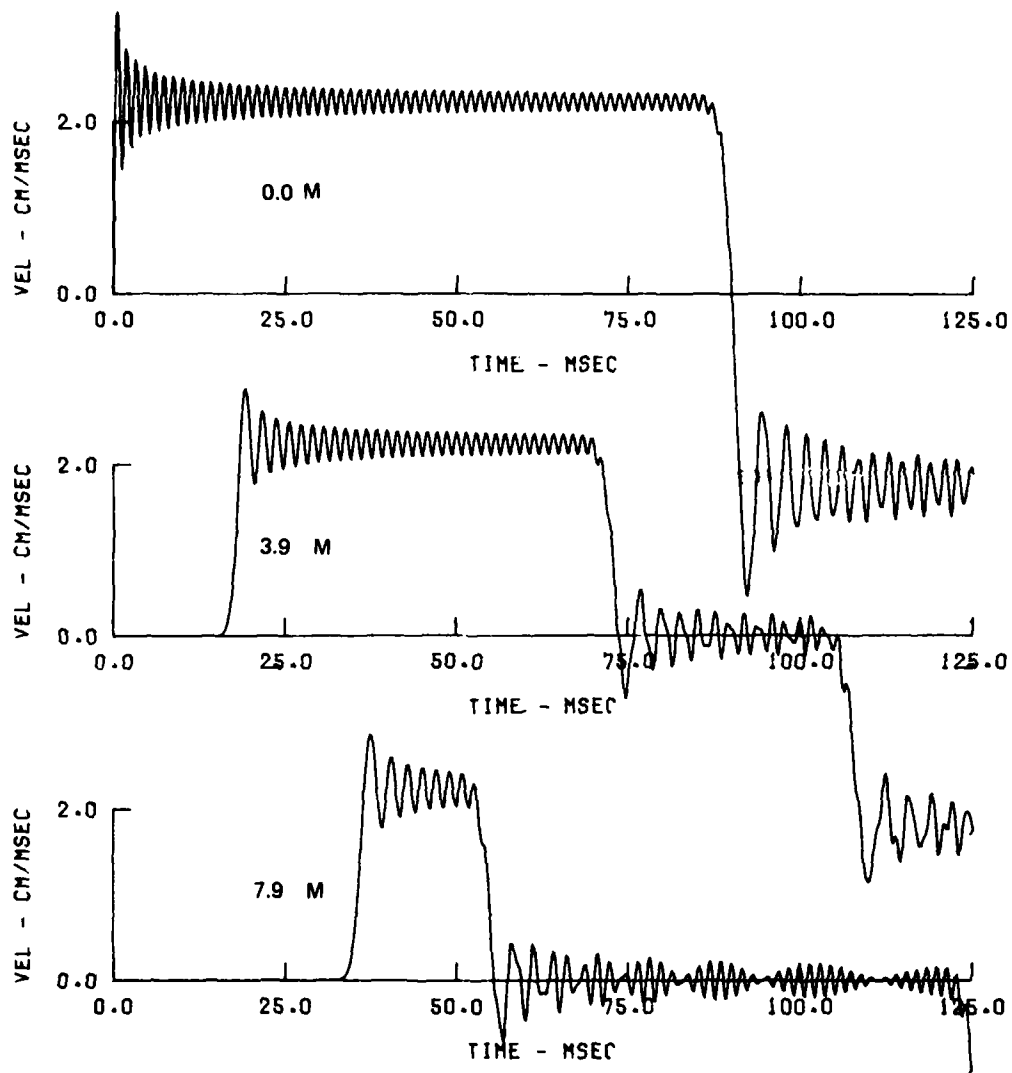


Figure 13. Velocity-time histories for Problem 3

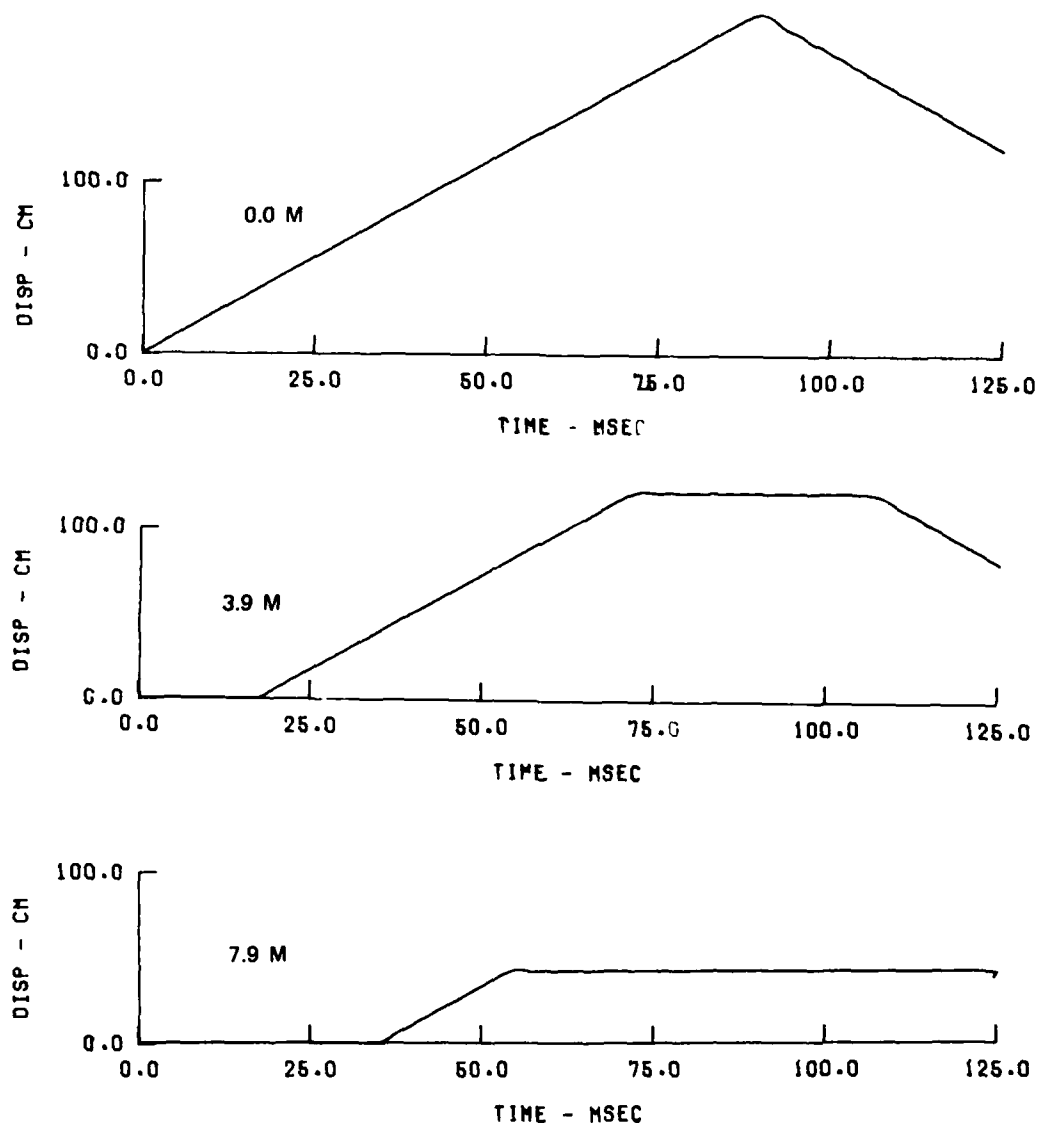


Figure 14. Displacement-time histories for Problem 3

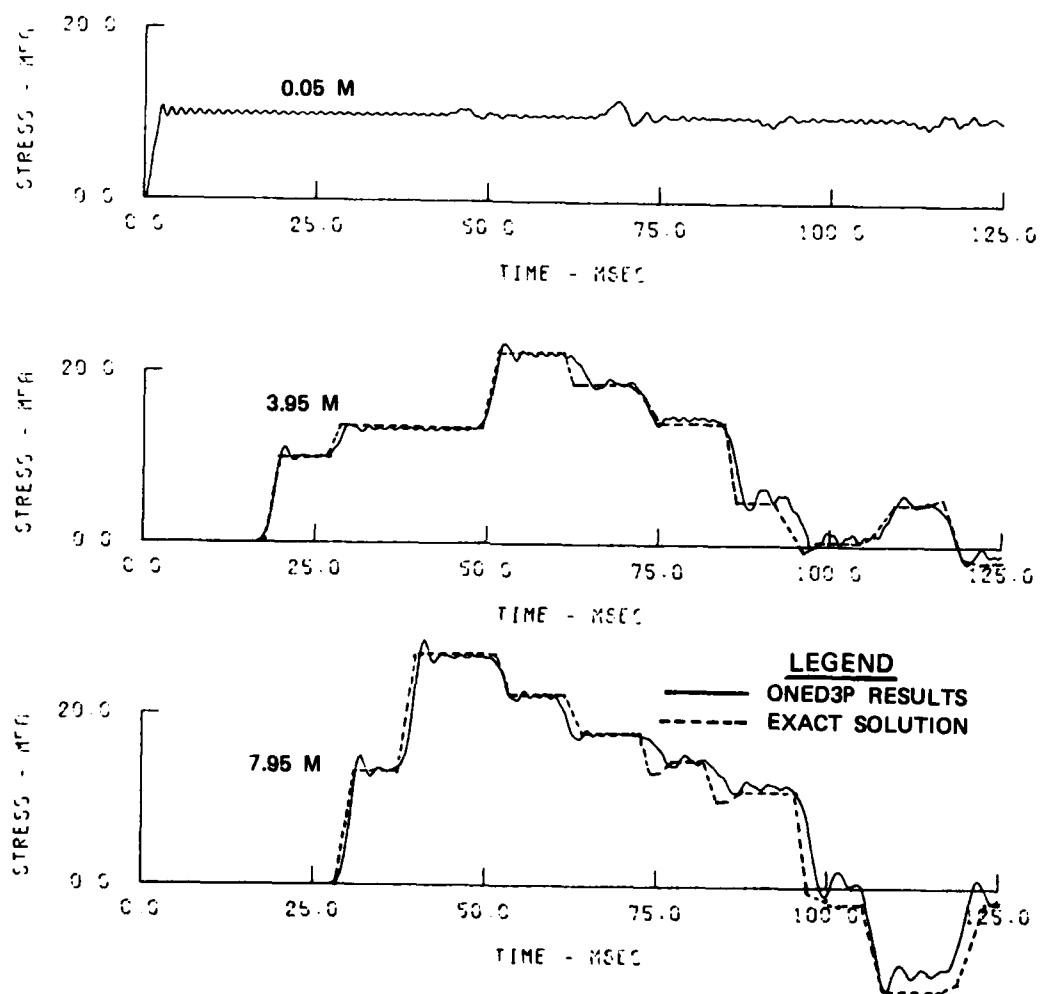


Figure 15. Stress-time histories for Problem 4



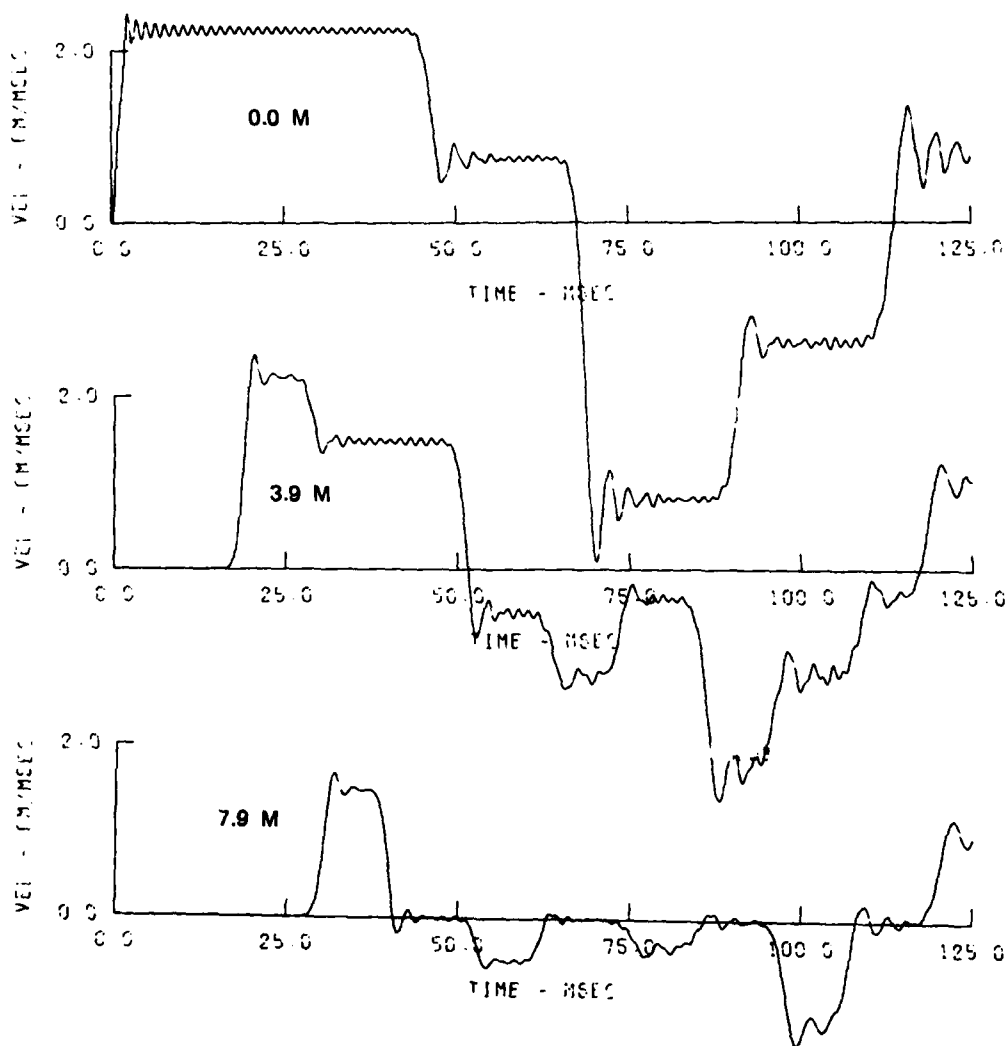


Figure 16. Velocity-time histories for Problem 4

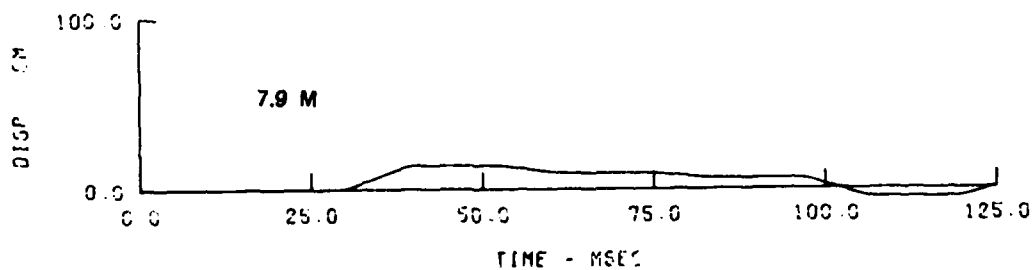
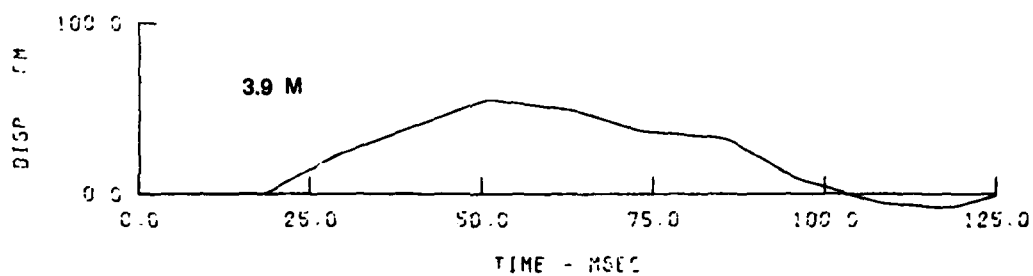
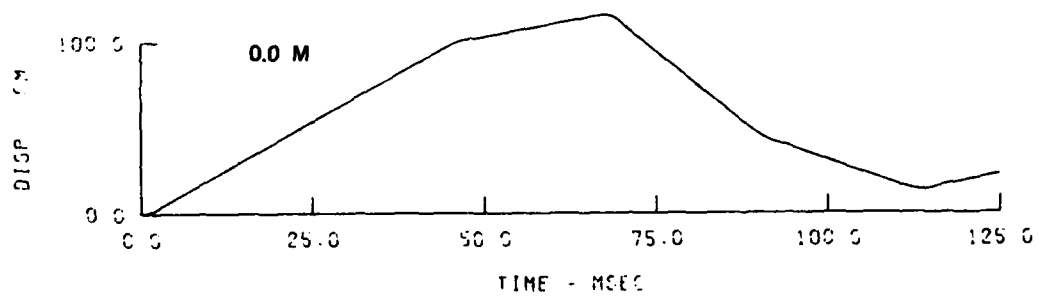
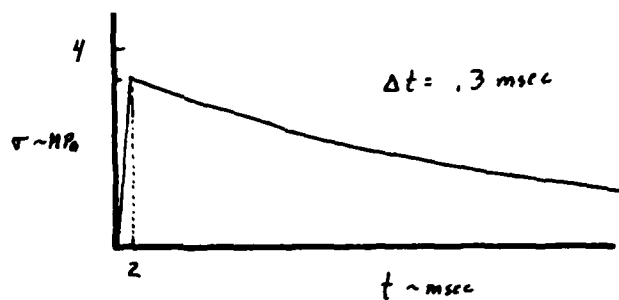
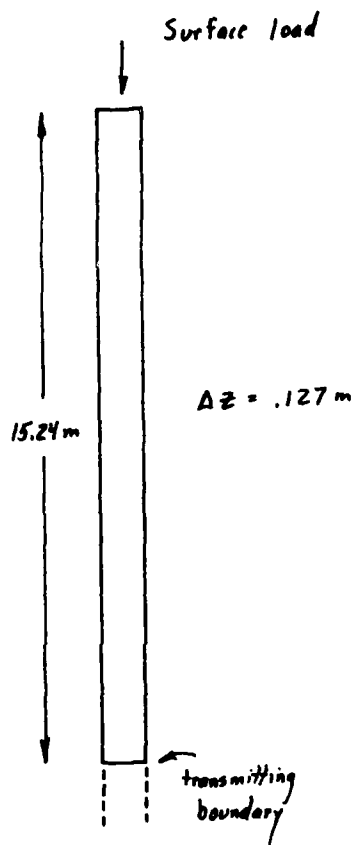
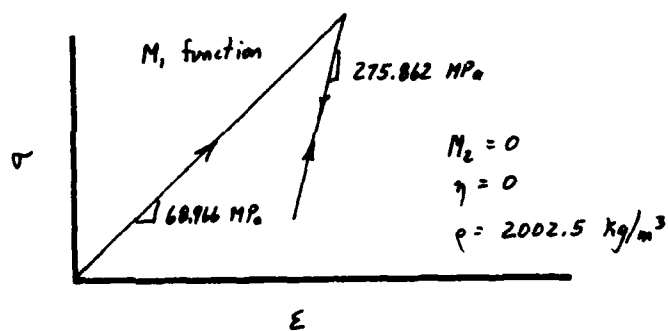


Figure 17. Displacement-time histories for Problem 4



b) SURFACE LOADING FUNCTION



c) MATERIAL PROPERTIES

a) PROBLEM GEOMETRY

Figure 18. Problem 5 description

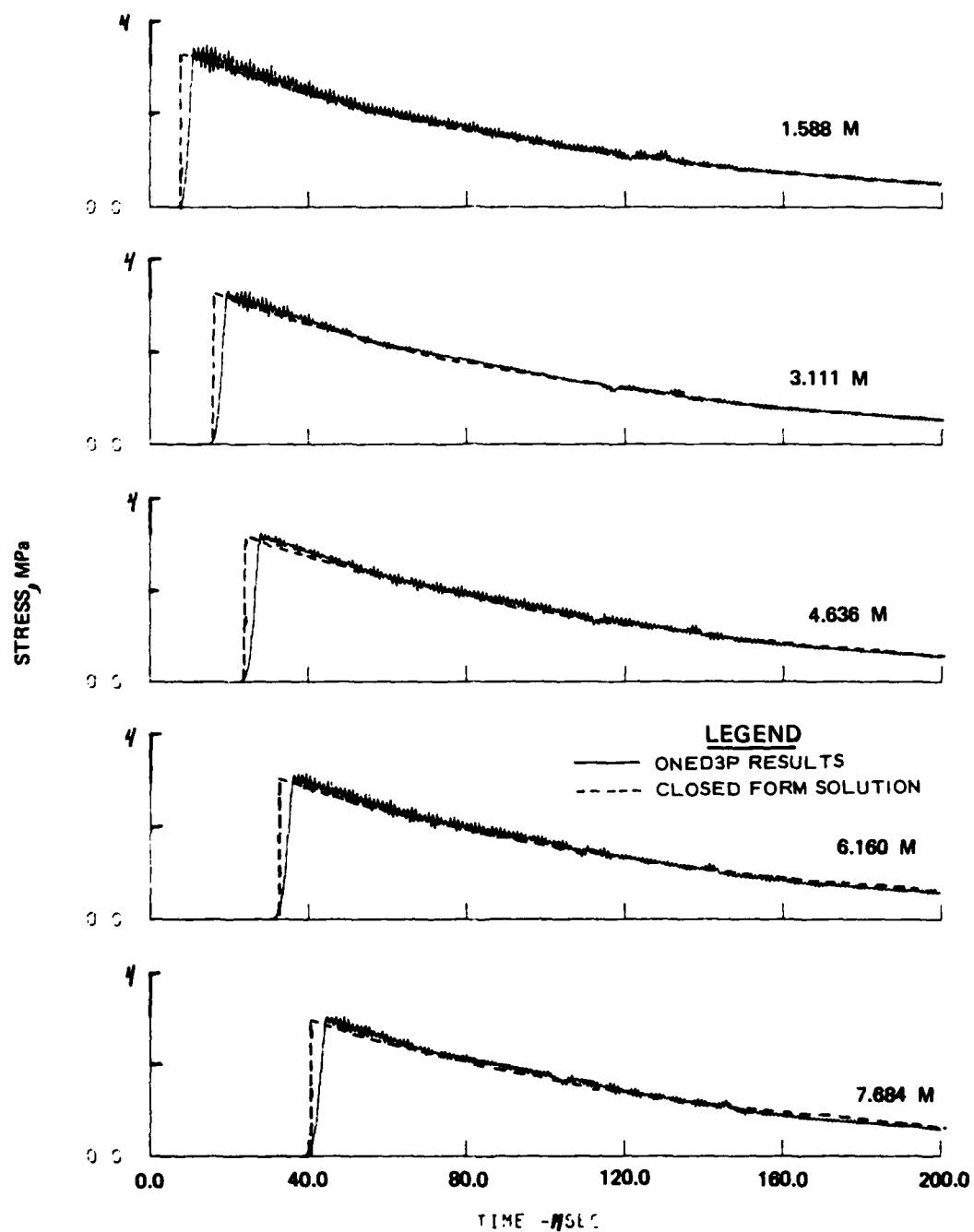


Figure 19. Stress-time histories for Problem 5

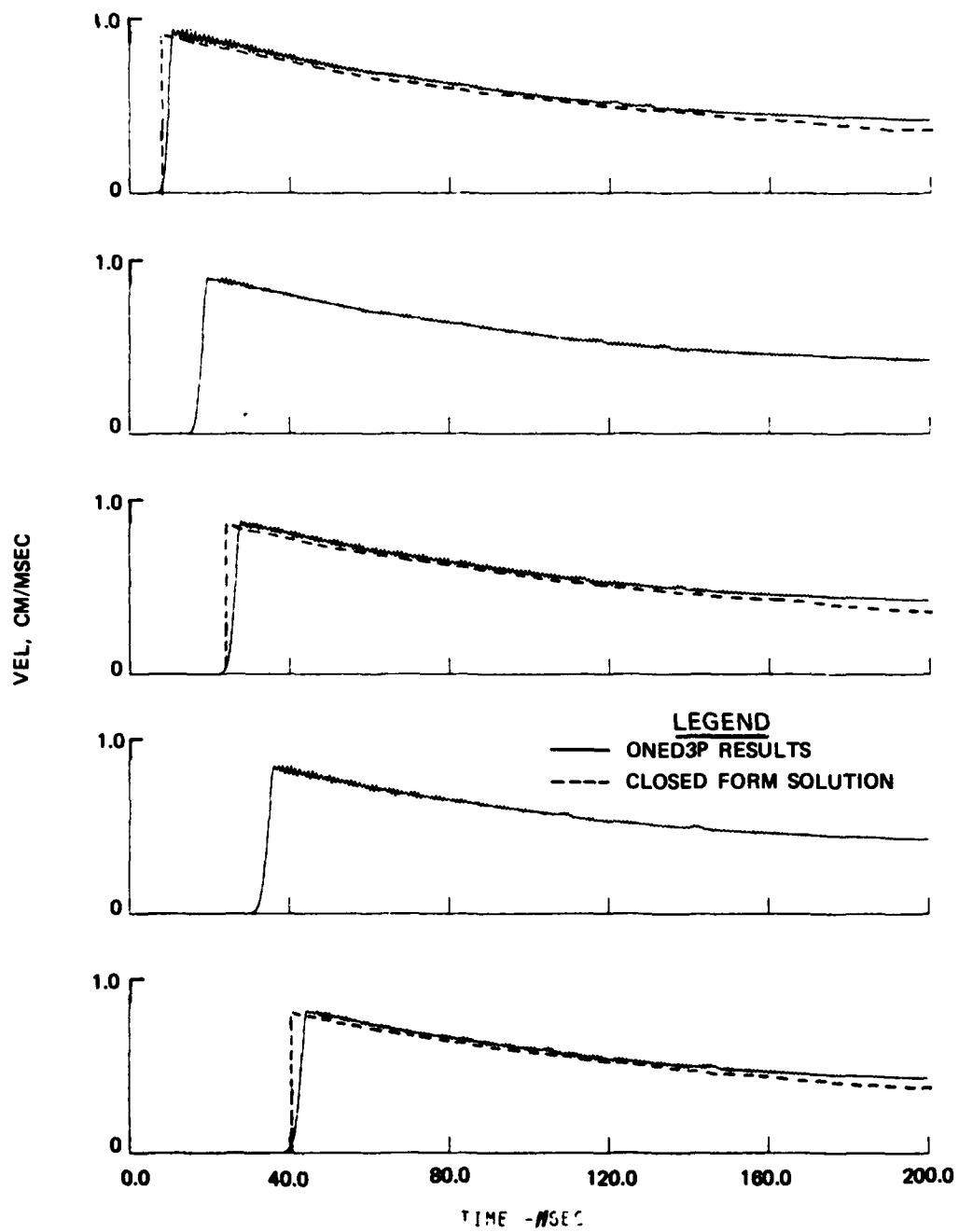


Figure 20. Velocity-time histories for Problem 5

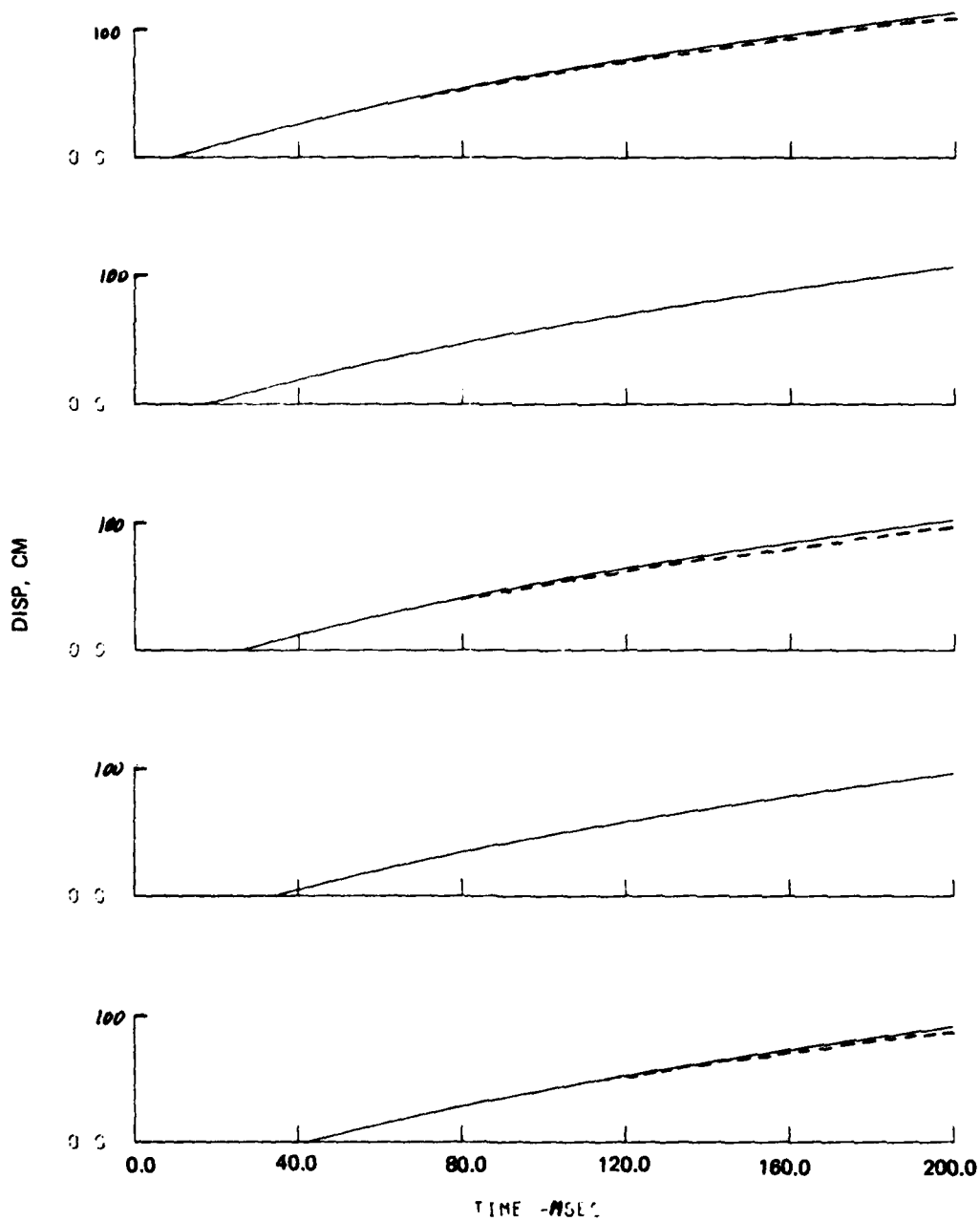


Figure 21. Displacement-time histories for Problem 5

Problem 6, is described in Figure 22 which also shows the data points used in both codes to digitize the nonlinear stress-strain curve.

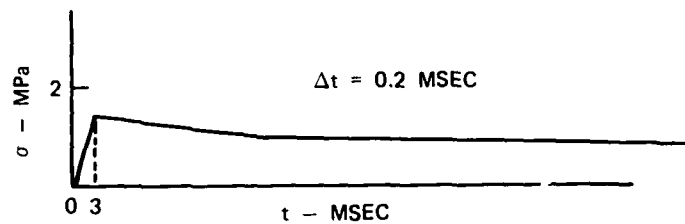
29. Stress-time histories from the two code calculations are compared at various depths in Figure 23. Obviously the first compression wave and its reflection look very much alike using either code. However, it appears that unloading waves which result in low stress levels travel much faster in the ONED3P calculation than in the ONED calculation. The reason for this lies within the unloading-reloading logic used by each code.

30. Consider Figure 24 which shows the unloading curves for any element as generated within each code. The ONED code calculates the stress level where an unloading curve bends as a percentage of the maximum previous stress computed in the element. Therefore, if the unloading curve was originally defined from the point A, then an unloading curve from point B would look like that shown in Figure 24a. On the other hand, the ONED3P was designed to account for the observation that many unloading curves bend at about the same stress level regardless of the stress value from which they originate. The resulting unloading curve from point B as computed by ONED3P is shown in Figure 24b. Comparing the two figures, it becomes obvious that if unloading takes place from stress level B to stress level C the slope of the unloading curve at C in the ONED3P calculation would be greater than the slope at C in the ONED calculation. Under these conditions, unloading waves in ONED3P would travel faster than similar waves in ONED.

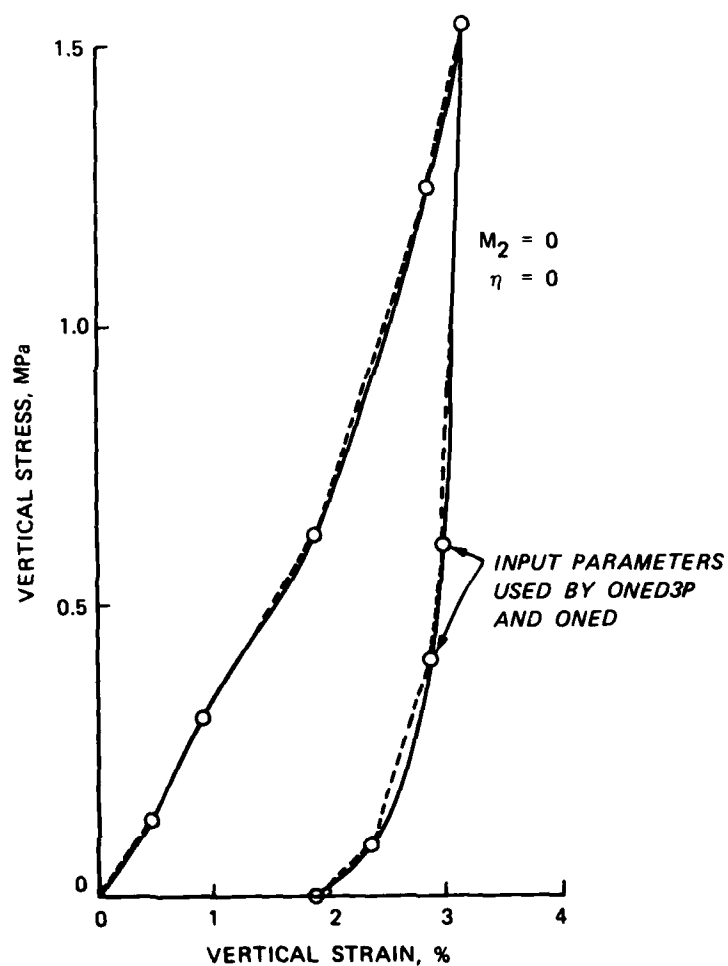
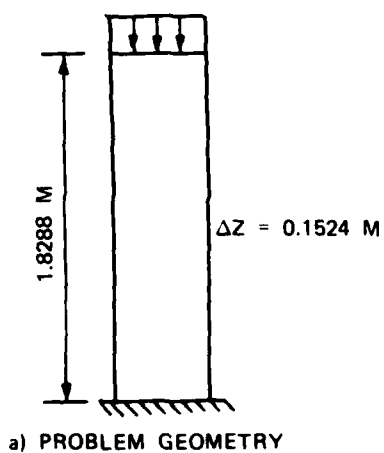
### Viscous Problems

#### Linear viscoelastic column

31. Using Laplace transform methods, Morrison<sup>9</sup> solved the problem of a semi-infinite column of linear viscoelastic material subjected to a step load at its surface. One of the material models he used was a three-parameter model like that of Figure 1a where the three parameters all had constant values. His results were presented in a nondimensional form of stress versus depth in the column at constant times. By



b) SURFACE LOADING FUNCTIONS



c) MATERIAL PROPERTIES

Figure 22. Problem 6 description



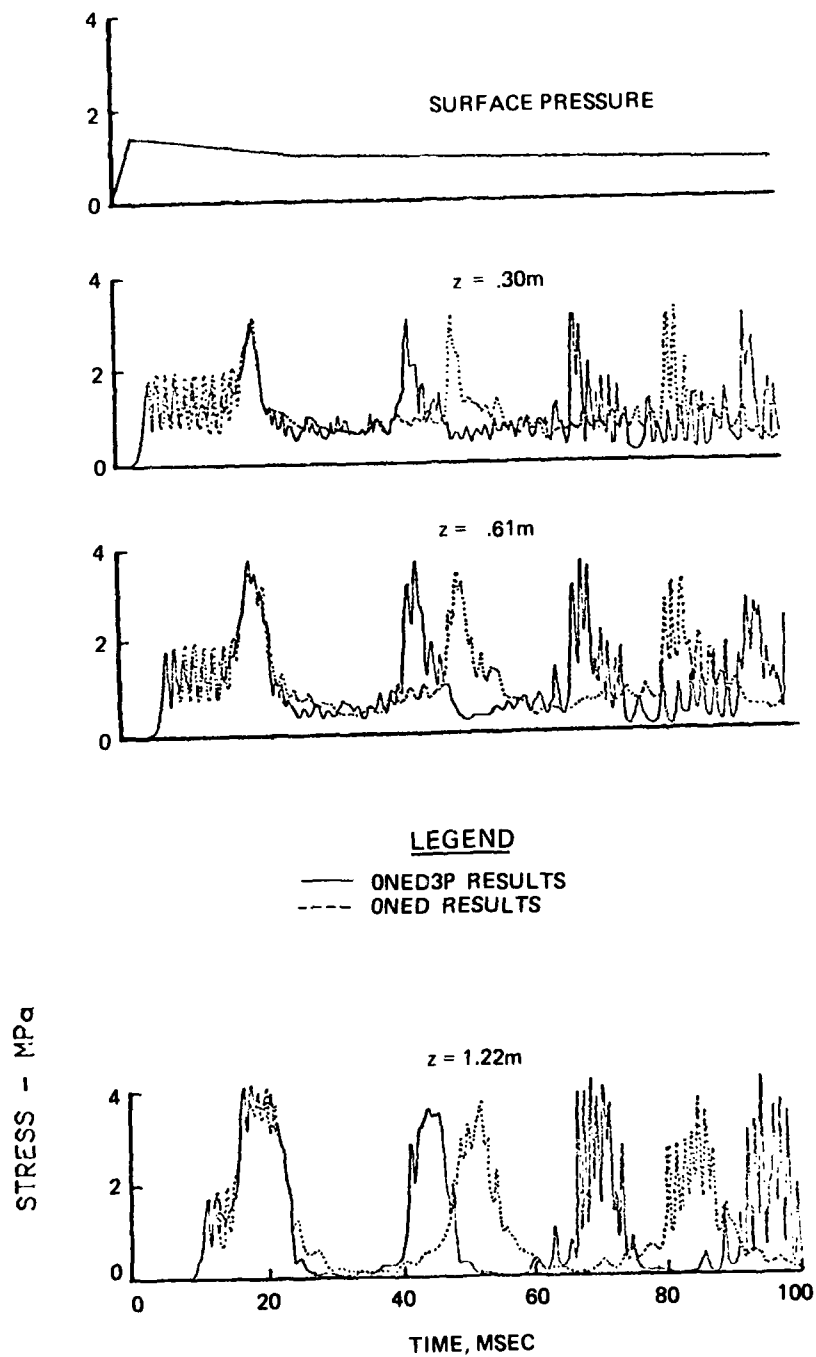


Figure 23. Stress-time histories for Problem 6

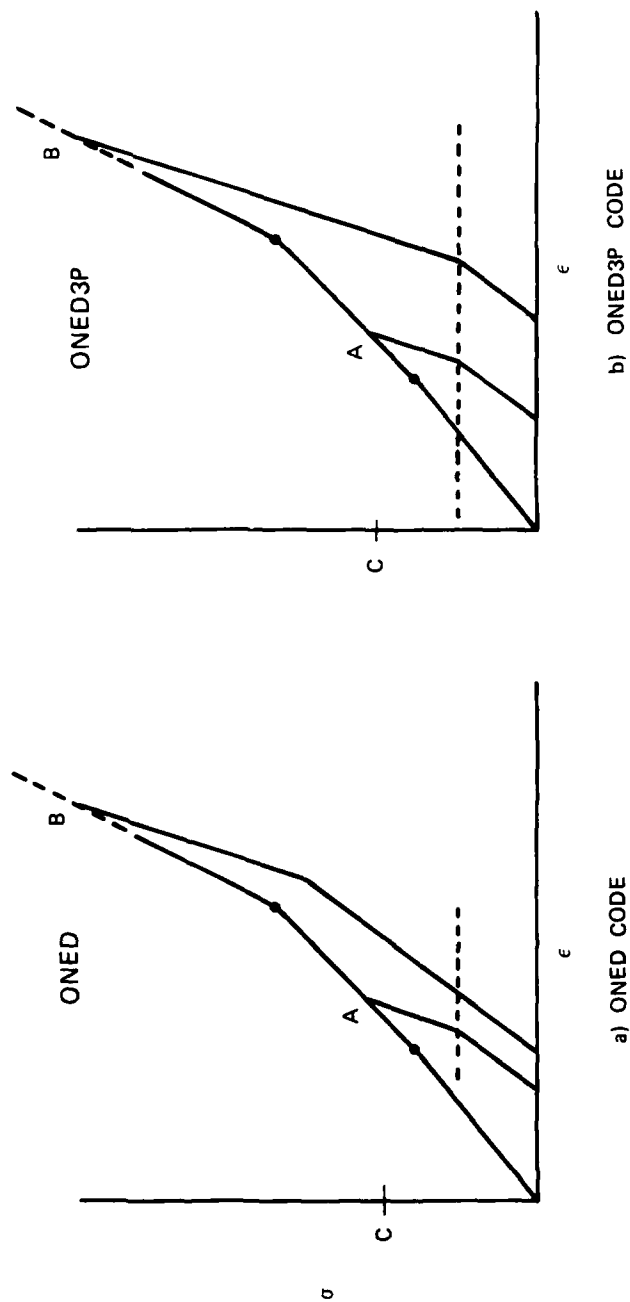


Figure 24. A comparison of ONED and ONED3P unloading logic

cross-plotting Morrison's results at given depths after interpolating additional constant time curves it was possible to generate stress-time histories from his published results which could then be compared with ONED3P results for any given problem.

32. Figure 25 describes the linear viscoelastic ONED3P calculations which were set up for comparison with Morrison's results for a given set of  $M_1$ ,  $M_2$ , and  $\eta$  values. The analytical results assumed a step load on the surface of the column, whereas a finite rise time was chosen for the ONED3P calculations. In fact, two rise times were selected: 1 msec for Problem 7 and 0.5 msec for Problem 8.

33. Results for each problem are shown as stress-time histories at various depths in Figure 26. Morrison's solution is shown as dashed curves. Although the finite rise times in each problem contribute to poor early-time comparisons at each depth, the late-time comparisons look very good. There was sufficient viscosity in these calculations to cause low-stress-level waves to travel at a speed determined by the sum of  $M_1$  and  $M_2$  and to arrive at each depth at the correct time (as predicted by Morrison's solution).

34. Cutting the rise time in Problem 8 caused an overshoot of calculated stress compared to Morrison's predictions. The reason for this phenomenon is not clear.

35. The question of how well the transmitting boundary works in viscous calculations cannot be answered in this section. The column of material in Problems 7 and 8 is long enough that for the selected material a stress wave would not reflect back from the bottom boundary to the 3-metre depth in 20 msec. Transmitting boundaries for viscous materials will be discussed later in paragraphs 44-46.

#### Effect of loading rate and viscosity on wave speeds

36. Consider a column of material which behaves like the three-parameter mechanical model shown in Figure 1a and which is loaded by a loading function with a finite rise time. Intuitively, as that rise time decreases, stress waves travelling through the column should travel

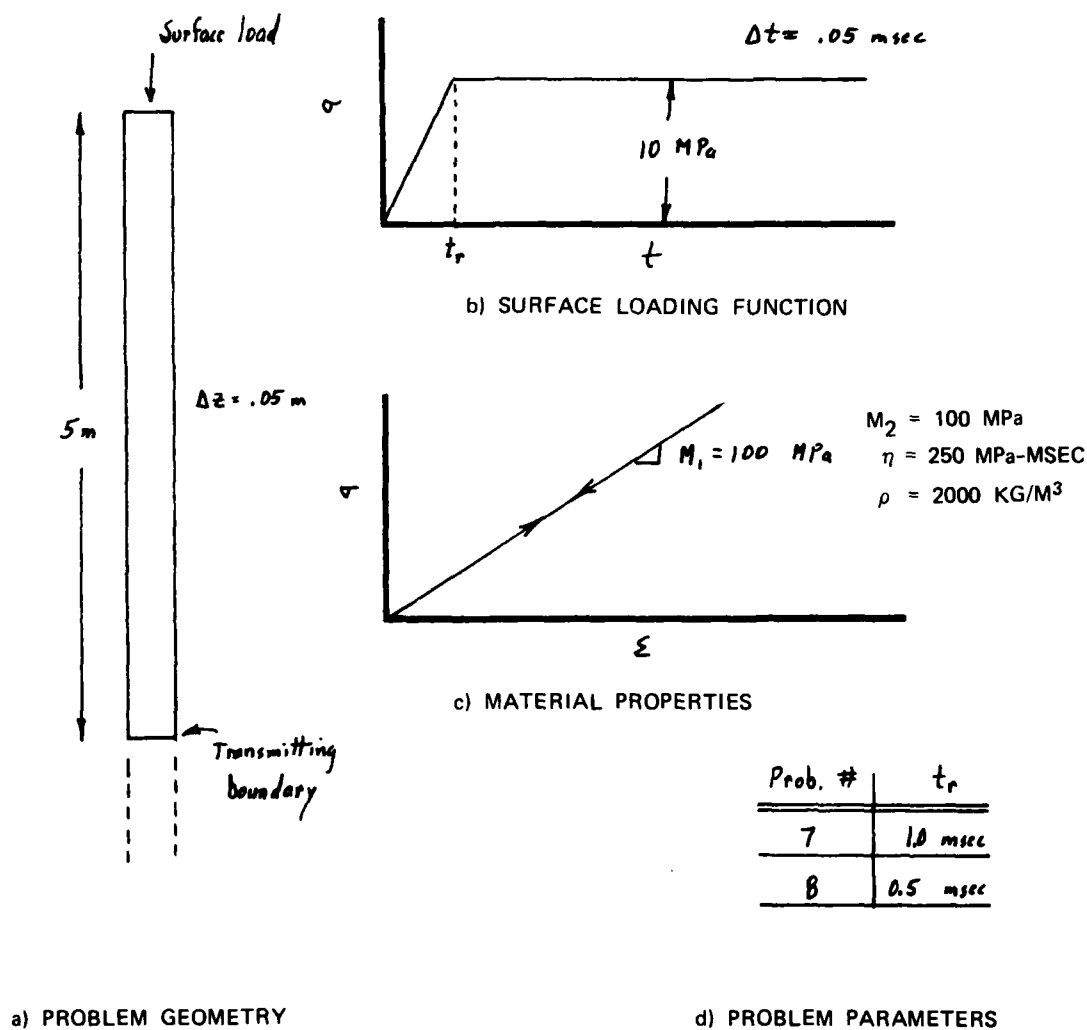
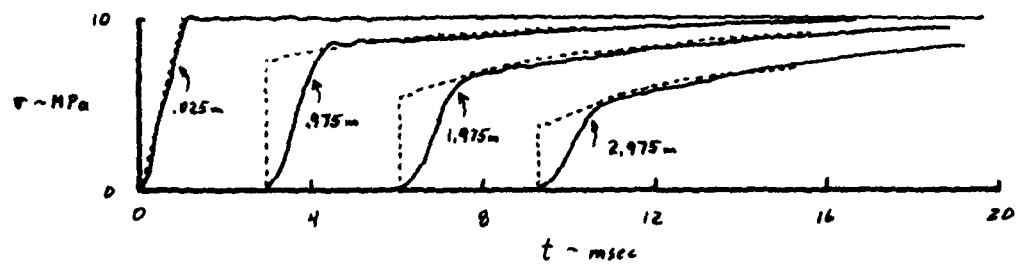
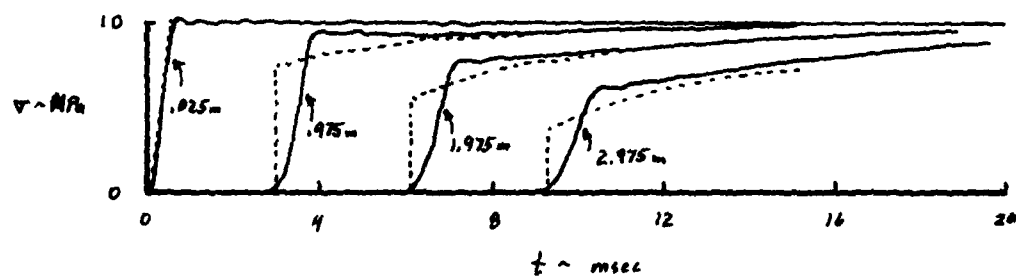


Figure 25. Description of problems for comparison with Morrison's solution



a) PROBLEM 7 --  $t_r = 1.0$  MSEC



b) PROBLEM 8 --  $t_r = 0.5$  MSEC

Figure 26. Comparisons of ONED3P results with Morrison's<sup>9</sup> solution

faster. The reason for this is that the viscous element or dashpot behaves more and more like a rigid element as the rate of loading on it increases which, in turn, magnifies the contribution of  $M_2$  to the total stiffness of the model. As this happens, the slope of the stress-strain curve at any point in the column will increase, which results in faster wave speeds.

37. Now consider a sinusoidal loading function acting on a three-parameter material having constant properties. From Kolsky<sup>5</sup> the speed of a sinusoidal dilational stress wave in such a material is a function of its frequency and may be written

$$C = \text{wave speed} = \left( \frac{2M_1(M_1 + M_2)}{\left[ \frac{(M_1 + M_2)^2 + M_1^2 \omega^2}{1 + \omega^2} \right]^{1/2} + \frac{M_1 + M_2 + M_1 \omega^2}{1 + \omega^2}} \right)^{1/2} \quad (13)$$

where  $\omega$  is a normalized frequency and is equal to the frequency of the wave ( $2\pi f$ ) times the "time of retardation" of a Kelvin-Voigt element ( $\tau$ ) which in this case is

$$\tau = \frac{\eta}{M_1 M_2 / (M_1 + M_2)} \quad (14)$$

Note that for very low frequencies,

$$C \rightarrow C_s = \sqrt{\frac{M_1}{\rho}}, \quad \omega \rightarrow 0 \quad (15)$$

where  $C_s$  is the slowest possible wave speed in the material and is associated with the element's quasi-static behavior whereas for very high frequencies

$$C \rightarrow C_{\max} = \sqrt{\frac{M_1 + M_2}{\rho}}, \quad \omega \rightarrow \infty \quad (16)$$

where  $C_{\max}$  is an upper bound on the wave speed and is associated with the parallel spring elements. Finally, combining Equations 13, 14,

and 15, one has

$$\frac{C}{C_s} = \left\{ \frac{2(M_1 + M_2)}{\left[ \frac{(M_1 + M_2)^2 + M_1^2 \omega^2}{1 + \omega^2} \right]^{1/2} + \frac{M_2 + M_1(1 + \omega^2)}{1 + \omega^2}} \right\}^{1/2} \quad (17)$$

Note that for a given set of  $M_1$  and  $M_2$  values,  $C/C_s$  is a unique function of the product of frequency and viscosity.

38. Equation 17 is an analytical tool which can be used to predict the speed of a sinusoidal stress wave in a three-parameter material as a function of its frequency and the properties of the material. The question is: Will stress waves calculated by ONED3P behave in the same way? A series of six ONED3P calculations were devised to answer that question. Those calculations are described in Figure 27. The significance of the parameter values and loading frequencies which were chosen will soon be apparent.

39. Two examples of ONED3P calculation results for these problems are shown in Figure 28. That the stress-time histories at each depth in the column are not smooth sinusoidal curves may be attributed to numerical approximations inherent in any finite difference or finite element code. Observe, also, that the conditions at any depth do not become steady-state until after approximately two stress cycles. All six calculation results exhibited similar behavior.

40. Focusing attention on the stress-time histories at the three greatest depths, the following method was applied to determine the wave speed,  $C$ , for each problem. Utilizing the common drafting technique for drawing parallel lines with two triangles, a line was chosen for each problem which best described the intersection of each stress cycle with the time axes at the three greatest depths. Naturally this was done only for stress cycles which occurred after the stress wave became steady. The inverse slope of this line, being the speed with which each stress cycle propagates along the column, was then divided by  $C_s$  from Equation 15 and the results were plotted in Figure 29 which contains the

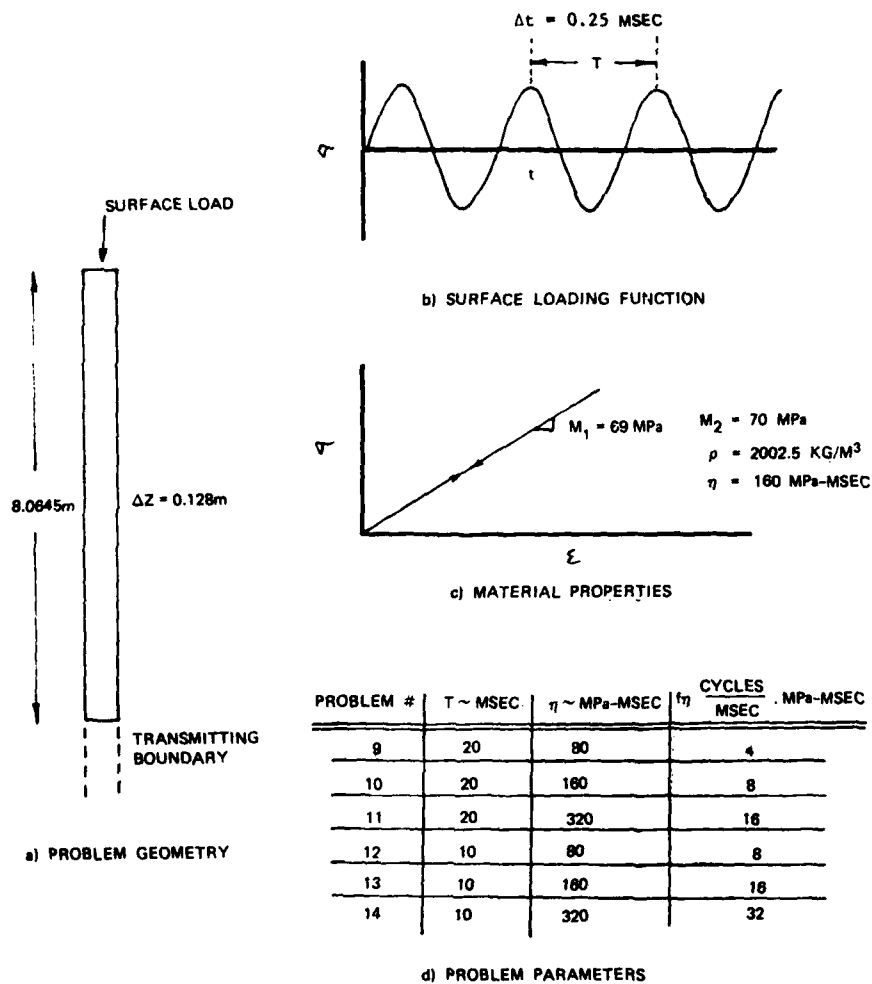


Figure 27. Description of problems used to study the effects of frequency of a sinusoidal stress wave on its wave speed



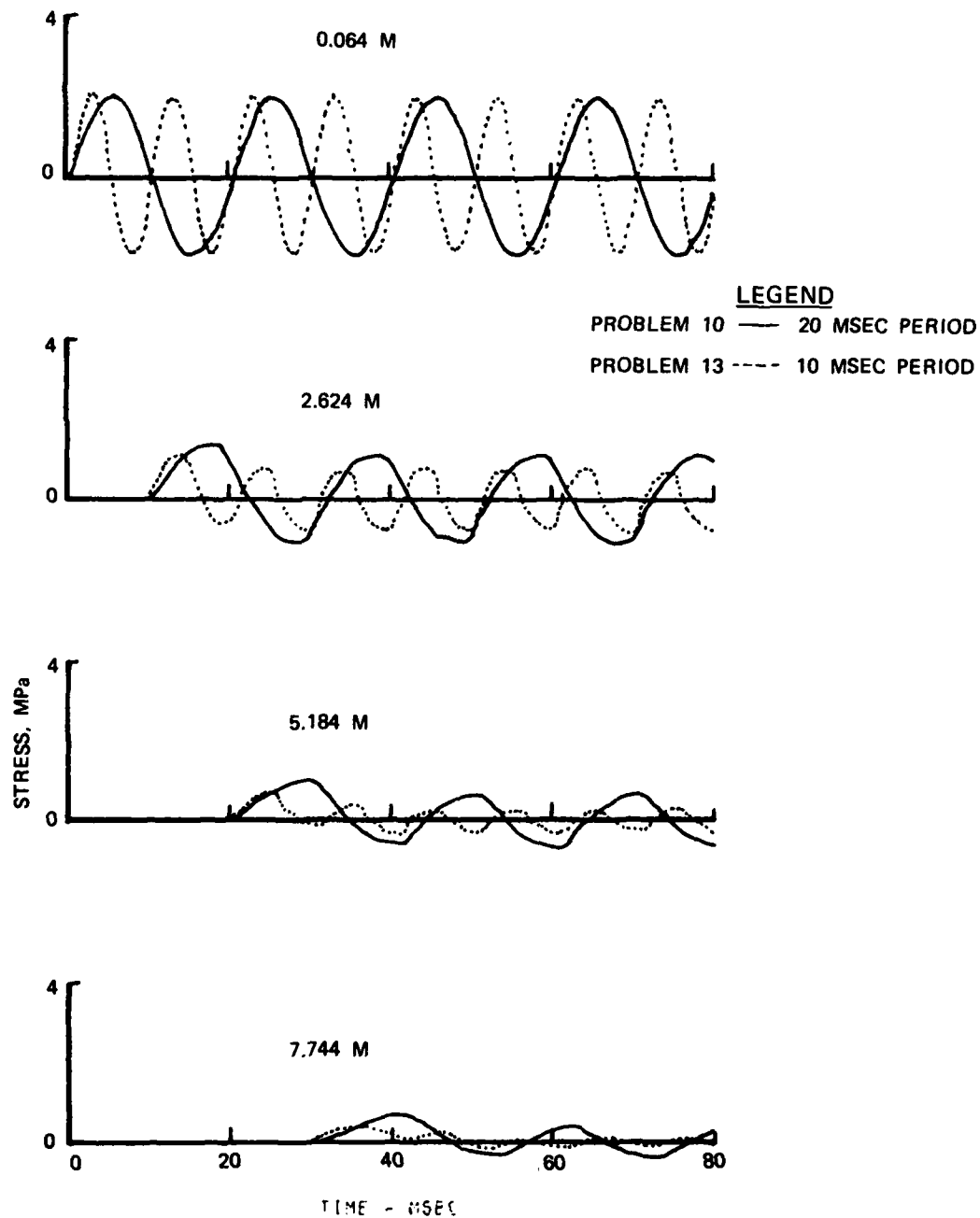


Figure 28. Stress-time histories for Problems 10 and 13

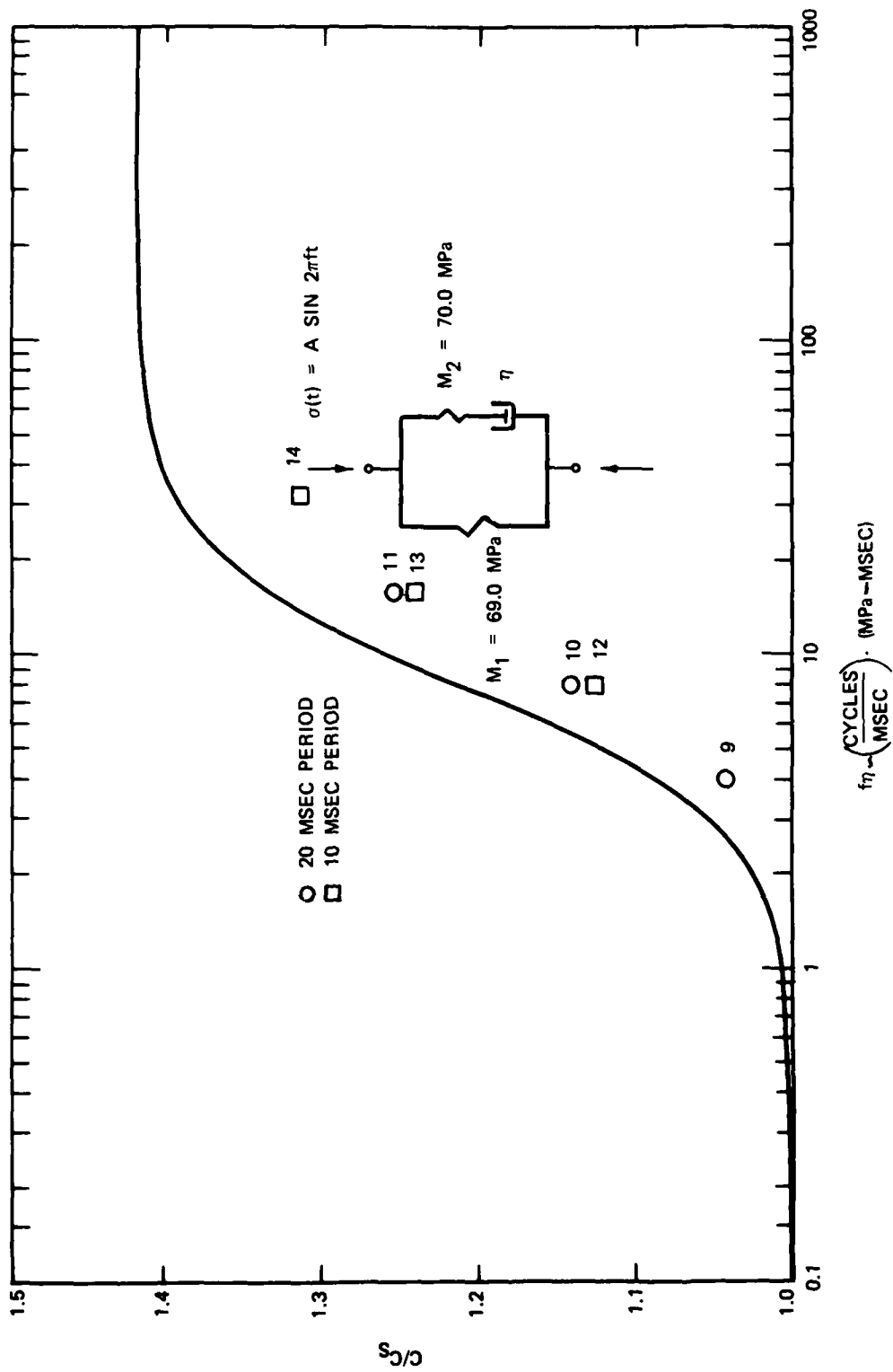


Figure 29. The variation of wave speed with frequency in a three-parameter linear viscoelastic solid

unique curve determined by substituting the  $M_1$  and  $M_2$  values for these six problems into Equation 17.

41. Qualitatively, the results of these six calculations were quite good in that they form points on a curve which, considering the nature of the ONED3P solution algorithm, is unique and which is nearly parallel with the predicted curve. Quantitatively, however, the calculated response is shifted to the right of the predicted response. This may be looked at in one of two ways: first, for a given frequency and viscosity, the calculated speed of a sinusoidal stress wave is less than would be predicted by theory, or, second, in order for a sinusoidal stress wave to travel at a given speed, it either must have a greater frequency than theory would require or it must be in a material which is more viscous than theory would dictate (or both). Time and funding do not permit a more thorough examination of these results. However, it is suggested that if the discrepancy between computed and predicted wave speeds is due to numerical approximations within the code, a calculation with a much finer grid and time step (and therefore more expensive) might result in a better correlation.

42. Although sinusoidal loading functions are analytically clean in the sense that they possess only one frequency component, the types of dynamic loading functions which are used in laboratory testing or which are observed in field experiments contain an infinite number of frequency components and may be approximated in many cases by a step load with a finite rise time such as that shown in Figure 30b. As a further exercise in studying the effects of viscosity on wave speed, the three problems described in Figure 30 were calculated and the resulting wave forms were plotted in Figure 31. Clearly, for a given rise time, increased viscosity results in faster wave speeds. The reason for this is shown in Figure 32 where it can be seen that increased viscosity causes a stiffer stress-strain response in a given element. If one took the rise time of the loading function as one-fourth of the period of a sine wave, it would be possible to calculate  $C/C_s$  values to be plotted in Figure 29. However, the initial arrival times for sinusoidal stress waves did not correlate well with theoretical predictions; neither do

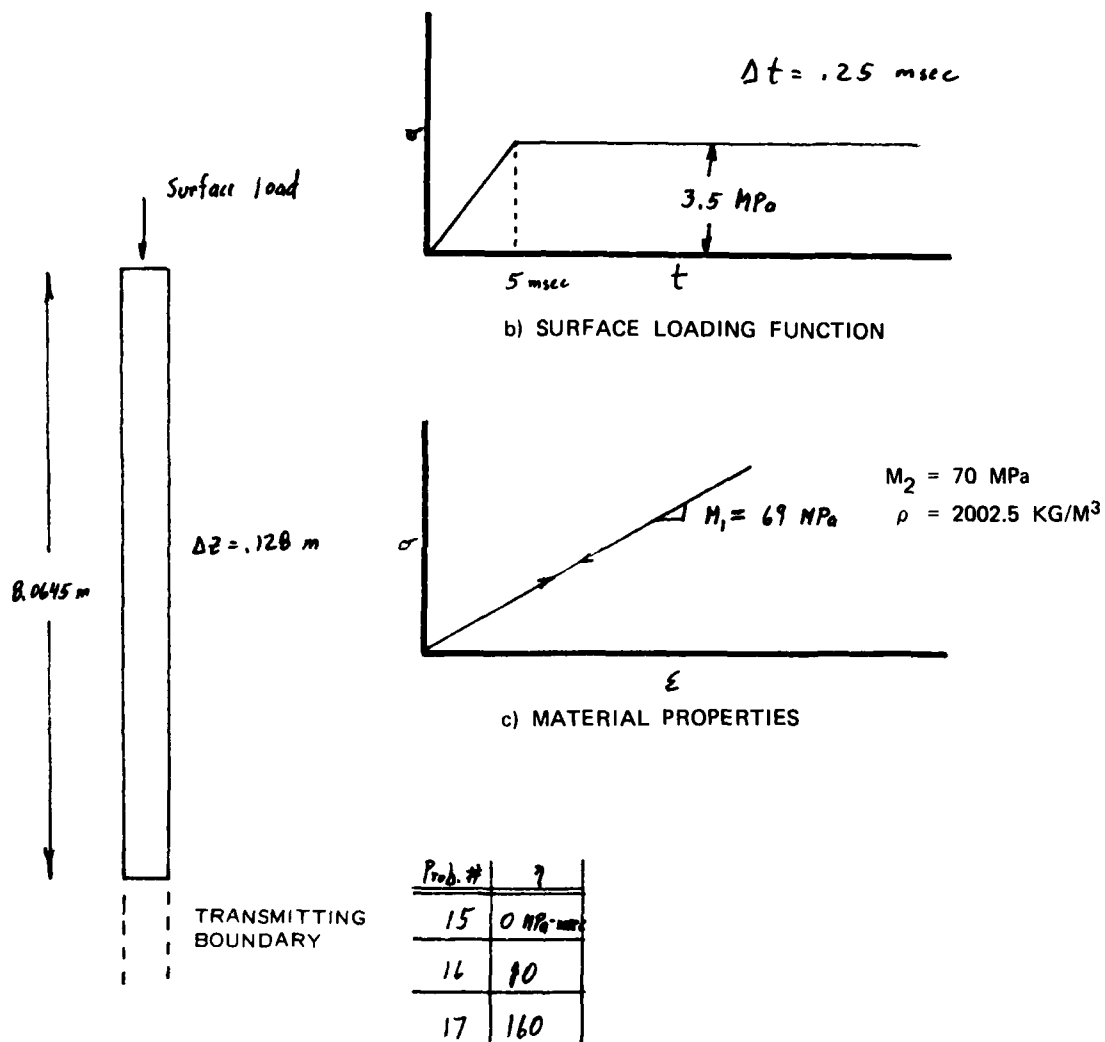


Figure 30. Description of problems used to study the effects of viscosity on wave speed

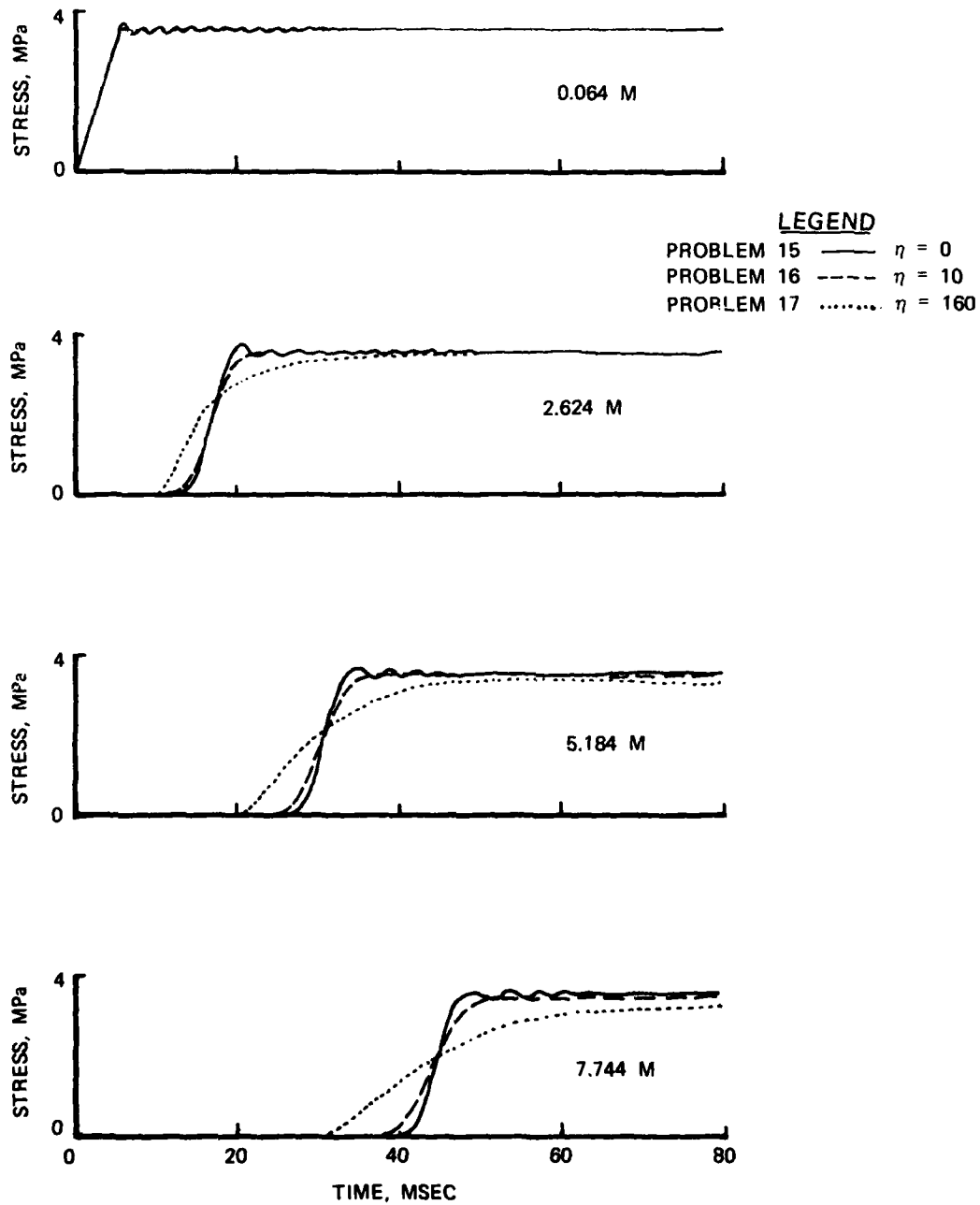


Figure 31. Stress-time histories for Problems 15, 16, and 17

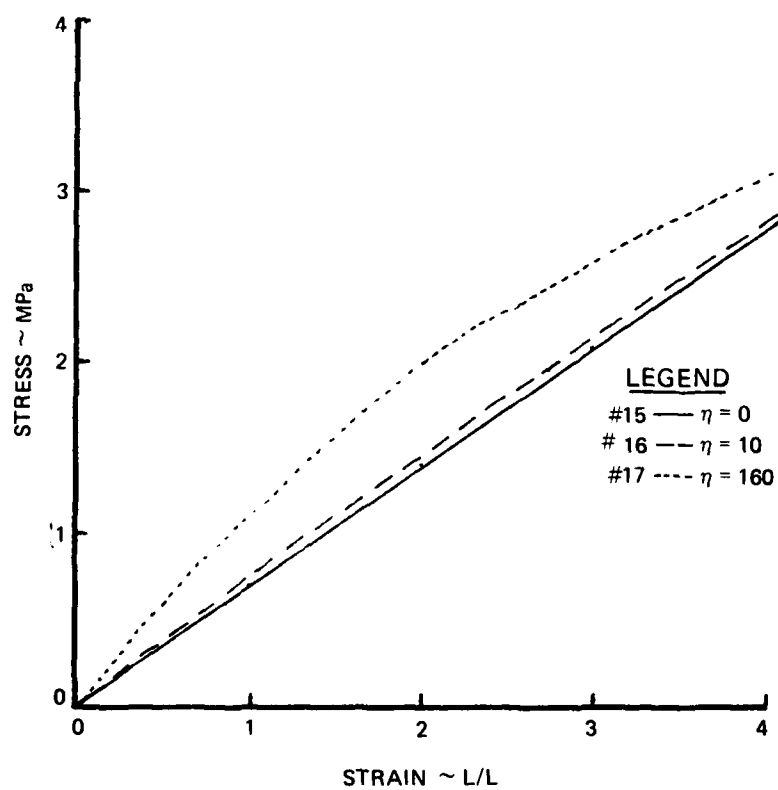


Figure 32. Stress-strain curves at a depth of 2.624 m for Problems 15, 16, and 17

the arrival times for Problems 15, 16, and 17.

43. At this time it is not recommended that Equation 17 be used to predict initial arrival times for stress waves generated by loading functions like that in Figure 30b. It is felt that the infinite frequency components in a finite rise time step load violate the assumptions under which Equation 17 was derived. However, Figure 31 does show clearly that wave speeds in a dynamic problem with a finite rise time are a function of viscosity. Furthermore, it will be demonstrated in Part V that, for a given viscosity, loading rates on the order of a fraction of a millisecond can affect wave speeds.

More on the transmitting boundary

44. The ONED3P transmitting boundary was shown to work quite well for the nonviscous linear hysteretic problem--Problem 5. As a check on how well it handles viscous materials, Problem 5 was recalculated with an  $M_2$  value equal to 69 MPa and three different  $\eta$  values (see Figure 18 for other problem parameters). These calculations were assigned  $\eta$  values equal to 10, 100, and 1000 MPa-msec, respectively.

45. Results, in the form of stress-time histories, are shown in Figures 33, 34, and 35. Assuming that the stress-time history at any depth should be a smoothly decaying function after the peak stress has been reached, it is obvious that some energy is reflected from the bottom boundary. For the highest viscosity the transmitting boundary appears to behave too stiff, resulting in a small compression wave being reflected back to the top of the column.

46. It is left to the discretion of the ONED3P user whether or not to use the transmitting boundary for his problem. These three calculations are only offered as examples, and it would be improper to draw from them general conclusions concerning all viscous problems.

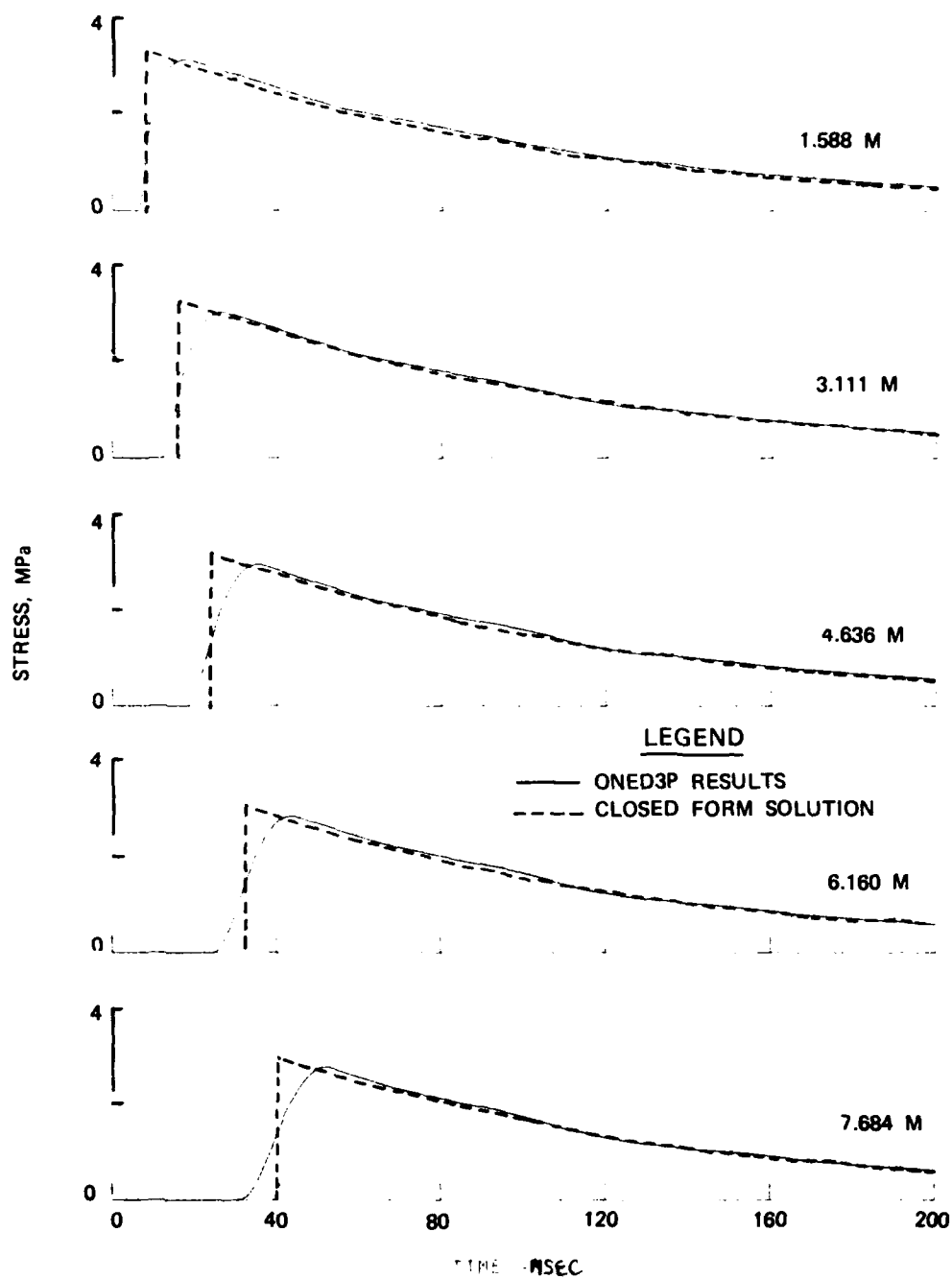


Figure 33. Stress-time histories for Problem 5;  
 $\eta = 10 \text{ MPa-msec}$ ,  $M_2 = 10,000 \text{ MPa}$



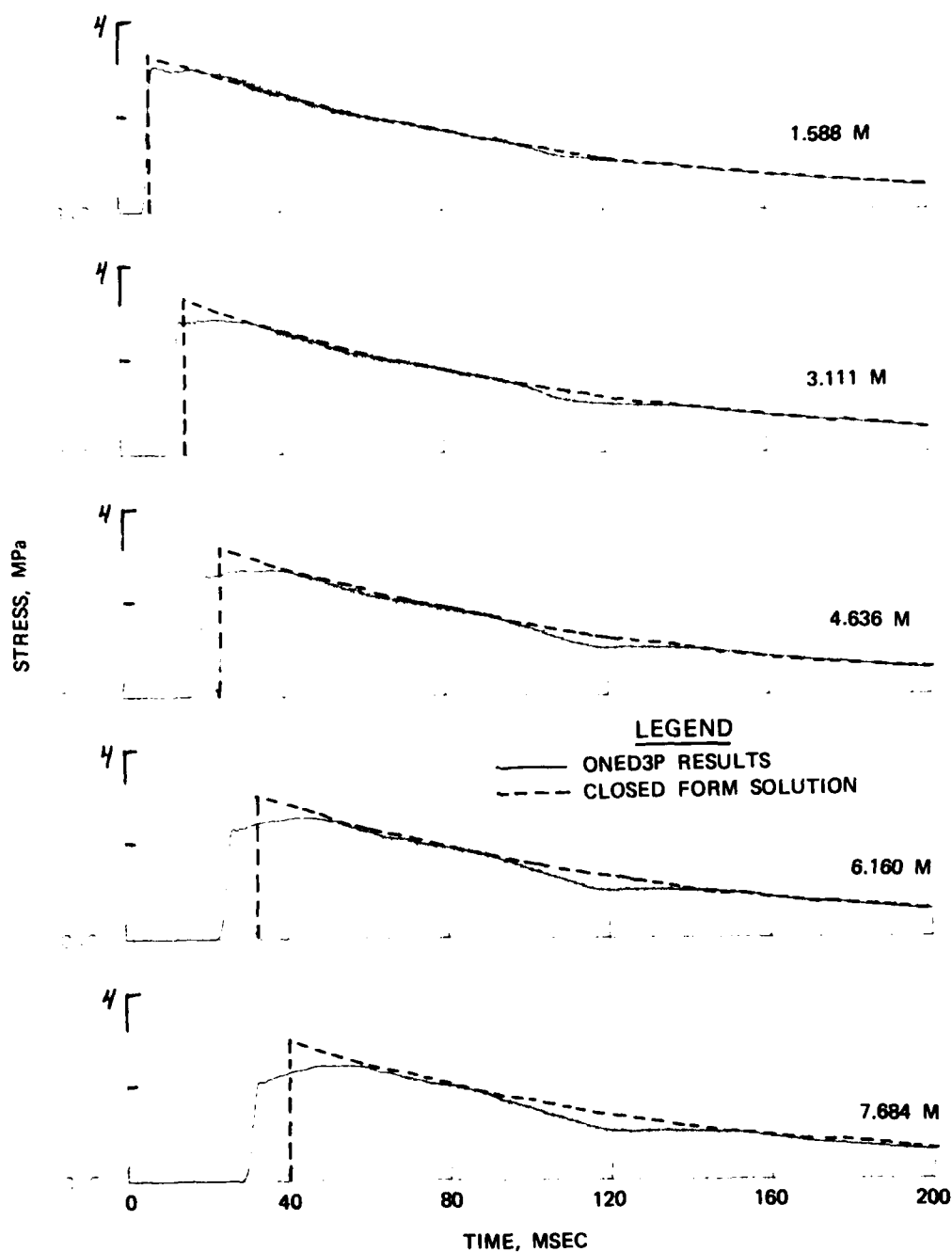


Figure 34. Stress-time histories for Problem 5;  
 $\eta = 100$  MPa-msec,  $M_2 = 10,000$  MPa

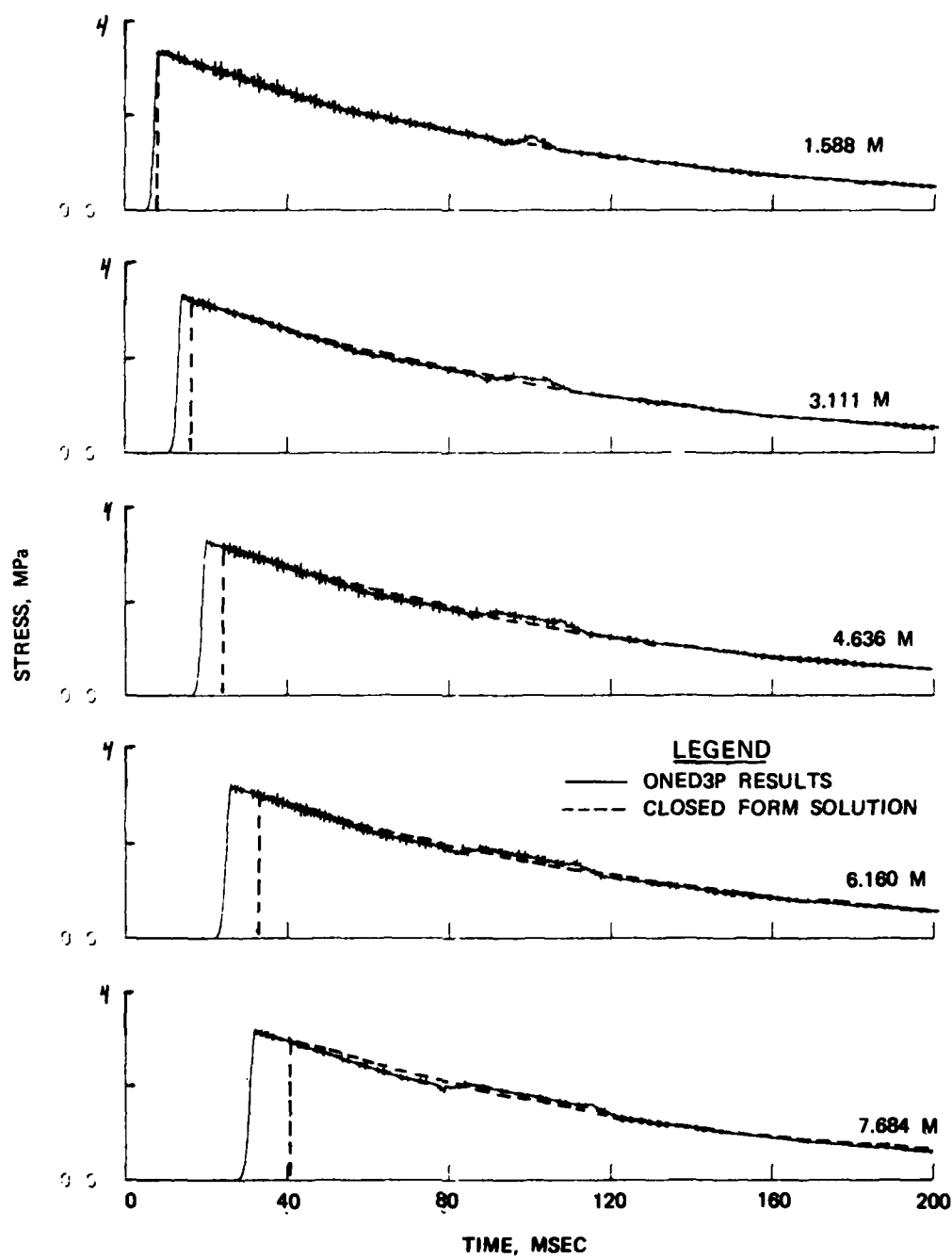


Figure 35. Stress-time histories for Problem 5;  
 $\eta = 1000 \text{ MPa-msec}$ ,  $M_2 = 10,000 \text{ MPa}$

## PART V: ONED3P SIMULATION OF FIELD EXPERIMENTS

### Background

47. Several field experiments have been conducted by WES personnel to test the response of shallow-buried, flat-roofed, concrete box structures to an approximate plane wave surface load generated by soil berm-covered explosions. The geometries of these experiments are presented in References 2 and 3. Soil stress histories at different depths were measured in the field during the events.

48. Laboratory material property tests were conducted on the sand backfill placed around and above these structures to determine the sand's compressibility. Rapid loading of sand backfill laboratory samples resulted in stiffer stress-strain responses than those obtained with static loadings. Field test results showed that the downward propagating stress waves travelled through the backfill sand with a speed much faster than the p-wave velocities determined from seismic refraction surveys. It has been suggested that this phenomenon can also be explained by the rate-dependent behavior of the sand.

49. An effort was previously made to simulate these field experiments with the ONED code using different rate-independent stress-strain curves to simulate the rate-dependent behavior of the sand backfill at different depths.<sup>10</sup> This brute-force method of accounting for rate dependence is at best an art which requires some posttest knowledge and is therefore not useful for pretest predictions. It would be both more convenient and physically more sound to use a true rate-dependent one-dimensional wave propagation code to try to simulate these field experiments. In this regard the development of ONED3P is very timely.

50. To demonstrate how ONED3P can be applied to real problems, backfill response in two of the above-mentioned field experiments, designated as FH4 and FH5, was simulated with the new code. Backfill properties were assumed to be the same in both experiments and only the surface loading functions were different.

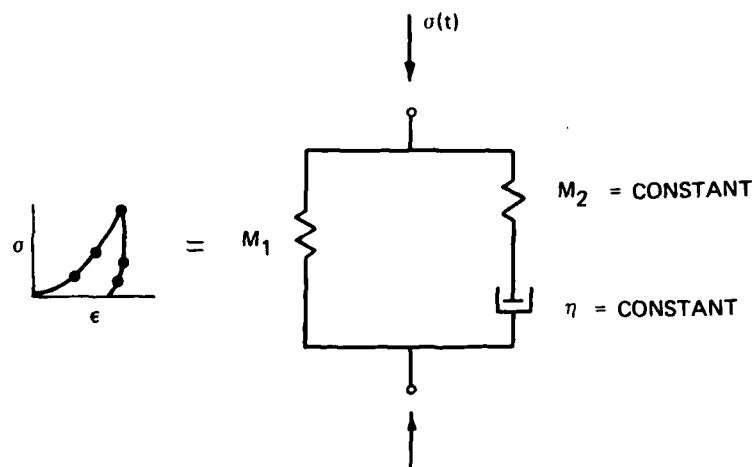
### Evaluation of Model Parameters

51. At present, the most sound method for determining quantitative values for the three mechanical parameters required by the ONED3P code is to select those parameters which will best reproduce both "static" and "dynamic" laboratory uniaxial strain test data. This can be done by using a simplified version of ONED3P called a driver. Put simply, the driver determines the strain response of the three-element model due to a known applied stress-time history.

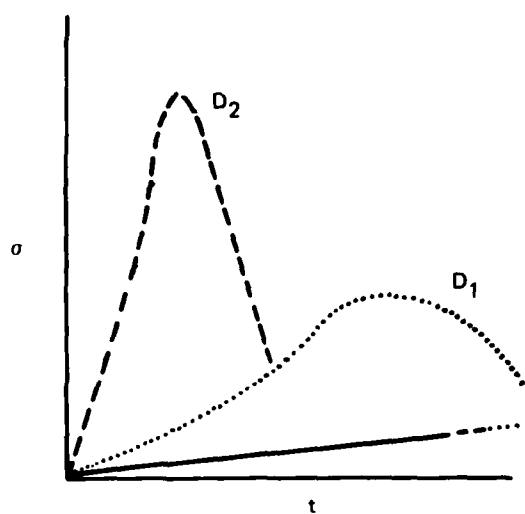
52. Consider the ONED3P mechanical model and typical laboratory data shown in Figure 36. In order to properly evaluate the model parameters, one must have a "quasi-static" response curve and at least two (but preferably more) "dynamic" stress-strain curves that will clearly depict the effects of rate of loading on the stress-strain response. The "dynamic" test data should include the fastest loading rates expected under field conditions. The "quasi-static" data can be derived from much slower loading rates; e.g., rise times on the order of tens of seconds are usually adequate. The three mechanical parameters will now be examined one at a time.

53. First there is the  $M_1$  function. It is defined by the "quasi-static" curve since very slow loading rates effectively cause the viscous and  $M_2$  spring elements in Figure 36 to disappear. Since ONED3P is an incremental code,  $M_1$  is, in fact, the tangent slope of the quasi-static curve at any point. Thus, for some nonlinear hysteretic materials,  $M_1$  may vary dramatically.

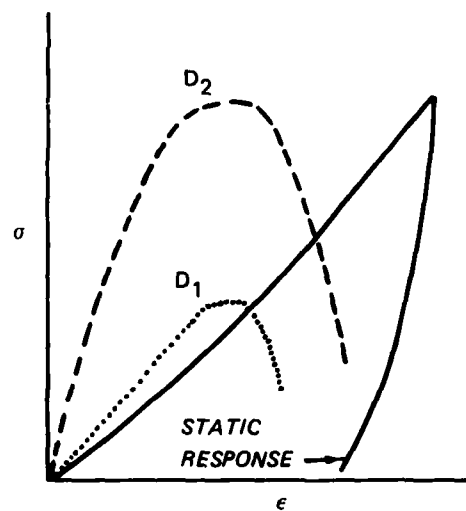
54. Next is the value of  $M_2$ , which in the current version of ONED3P is taken to be a constant. For extremely rapid loading rates the viscous element is nearly rigid and the response of the three-parameter model is governed by the sum of  $M_1$  and  $M_2$ . Thus, one possible means of establishing  $M_2$  is to determine the value of the initial tangent slope of the stiffest dynamic stress-strain loading curve and call that  $M_1 + M_2$ . This tangent should form an upper bound to all of the available dynamic stress-strain data.  $M_2$  can then be calculated because the value of  $M_1$  at the origin is known.



a) THREE-PARAMETER ONED3P MODEL



b) UX TEST: APPLIED STRESS PULSES



c) UX TEST: RESPONSE DATA

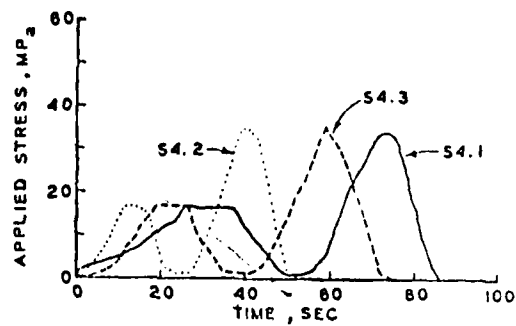
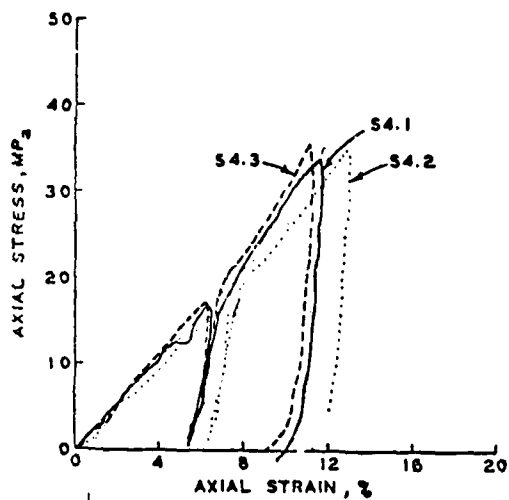
Figure 36. The three-parameter ONED3P model and typical laboratory uni-axial strain test data that can be used to determine quantitative values of the model parameters

55. Finally there is the viscosity coefficient,  $\eta$ , which, again, is presently taken to be a constant in ONED3P. Since  $\eta$  is effectively zero under quasi-static loading conditions and is effectively equal to  $\infty$  under almost instantaneous loading conditions, the task is to find the most appropriate value of  $\eta$  (between zero and  $\infty$ ) that will produce agreement with the available "dynamic" stress-strain data. This can be done by a trial-and-error process (once  $M_1$  and  $M_2$  are specified). That process entails varying  $\eta$  and executing the driver with each dynamic forcing function associated with the laboratory tests.

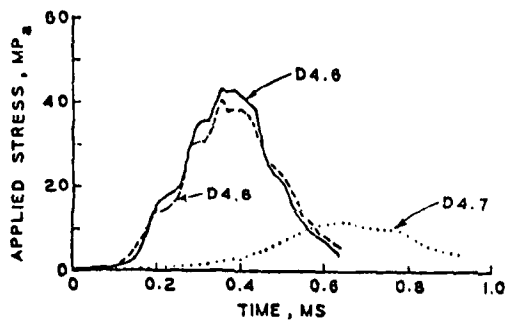
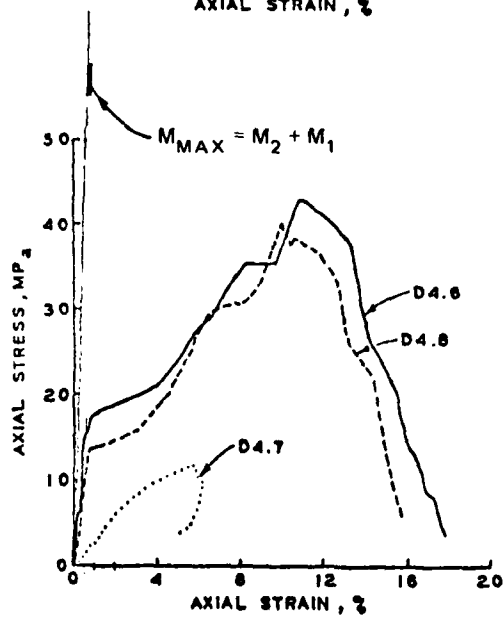
56. For example, consider the laboratory uniaxial strain test data derived from tests on samples of sand backfill used in the FH4 field experiment. These data are shown in Figure 37. The "static" response of the FH4 sand (Figure 37a) is well behaved, and a bilinear approximation to the average of the loading curves was assumed for  $M_1$  while the unloading response was assumed to be a straight line. The change in slope of the loading curve occurred at 20 MPa and 7.6 percent strain, while the unloading curve was drawn from 35.2 MPa and 11.8 percent strain to 0 MPa and 10.8 percent strain.

57. As for  $M_2$ , the steep tangent shown in Figure 37b was chosen at the upper bound to the data; its slope was such that  $M_2$  was calculated to be approximately 10,000 MPa.

58. Finally, several values of  $\eta$  were arbitrarily chosen and the driver was exercised with the applied load functions from laboratory tests D4.6, D4.7, and D4.8, which are all shown in Figure 37b. A value of  $\eta = 10$  MPa-msec was finally selected. It gave a good approximation to the D4.7 dynamic stress-strain curve but significantly undercut the initial slopes of the faster tests (D4.6 and D4.8) as shown in Figure 38. Before discussing the ONED3P results using these parameter values it should be noted that another possible method for establishing the material parameters is as follows. Given some  $M_1$  or "quasi-static" function,  $M_2$  and  $\eta$  could be derived through iterative ONED3P calculations against experiments like FH4 such that the arrival times of the stress wave fronts eventually match those measured in the field tests. Other

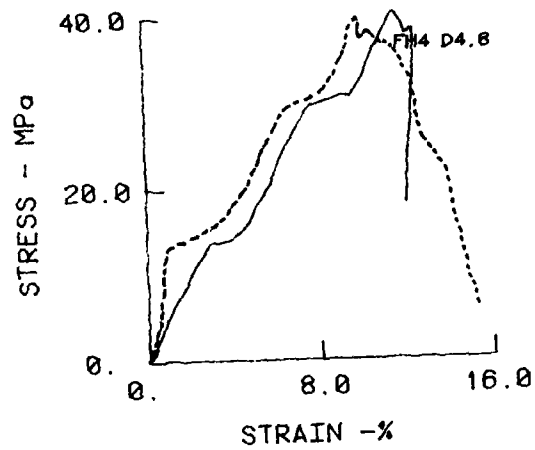


a. "QUASI-STATIC" TEST RESULTS



b. "DYNAMIC" TEST RESULTS

Figure 37. Laboratory uniaxial strain test results for FH4 backfill sand



**LEGEND**  
 — ONED3P DRIVER  
 - - - LAB MEASUREMENTS

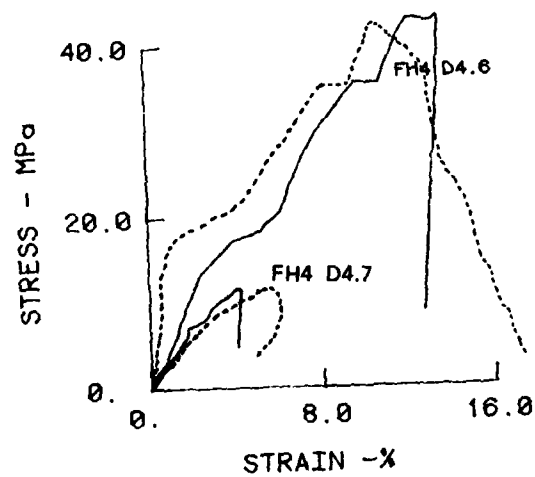


Figure 38. A comparison of the three-parameter model fit with dynamic FH4 laboratory results



specific problem needs not covered herein could dictate additional approaches to determining the ONED3P parameters.

#### Calculation Results

59. Two ONED3P calculations were set up for comparison with FH4 and FH5 field test results. Sand density was taken as  $1685.3 \text{ kg/m}^3$ . A grid spacing of 0.3175 cm was used and the time increment was taken to be 0.001 msec. In fact, the problem set-ups were identical to the previously mentioned ONED code simulations of the same tests. (ONED3P ran 30 percent faster than ONED for each calculation.)

60. Figures 39 and 40 compare stress-time histories generated at various depths by ONED3P with the measured field stress wave forms. Also included on those figures are the dynamic stress-strain curves generated by ONED3P at the corresponding depths. Although these simulations are only reported as an example of how to apply ONED3P to a real problem, a brief discussion of these results is still warranted.

61. First it is obvious that the calculated wave forms are different than the measured wave forms. There are at least two things which could be done to improve that comparison. One would be to obtain a better fit to the dynamic laboratory data in the previous section using the model in its present form. However, there is presently not enough flexibility in the model parameters to preserve both the low- and high-stress stress-strain responses shown in Figure 38. On the other hand, it is very likely that further developments in ONED3P might result in a better simulation of the measured wave forms. For example, if viscosity increased with stress level, a sharper wave front would be calculated. Such modifications to ONED3P are being considered but are not within the scope of this report.

62. On a more positive note, the stress wave arrival times at various depths were calculated with greater success. Note that the calculated stress waves slow down as they travel deeper into soil. The observation is consistent with the dynamic stress-strain behavior of each element. As the wave travels deeper into the soil, the dynamic

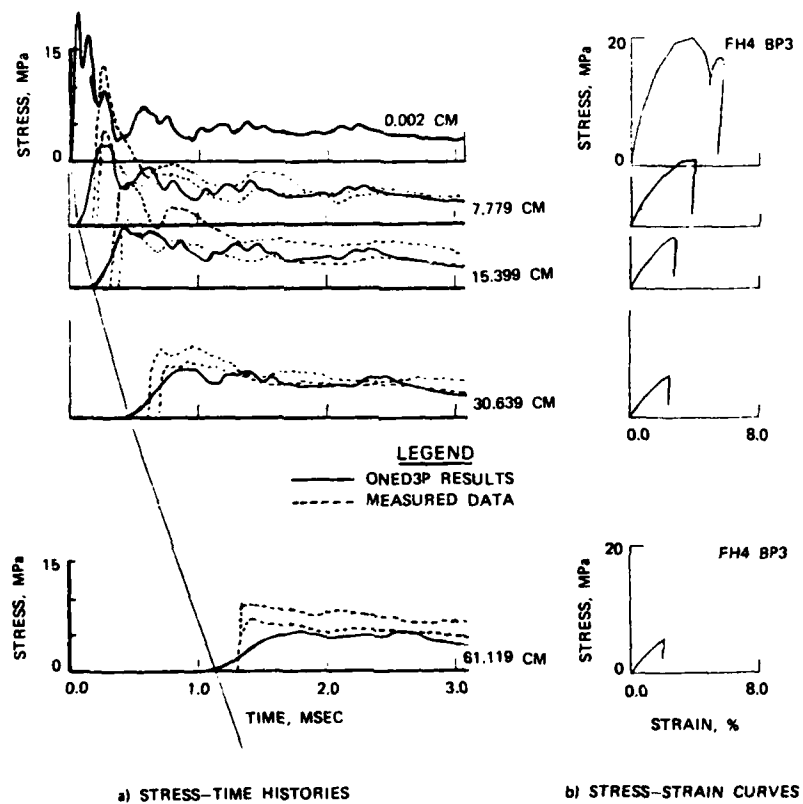


Figure 39. A comparison of ONED3P results with FH5 field test measurements

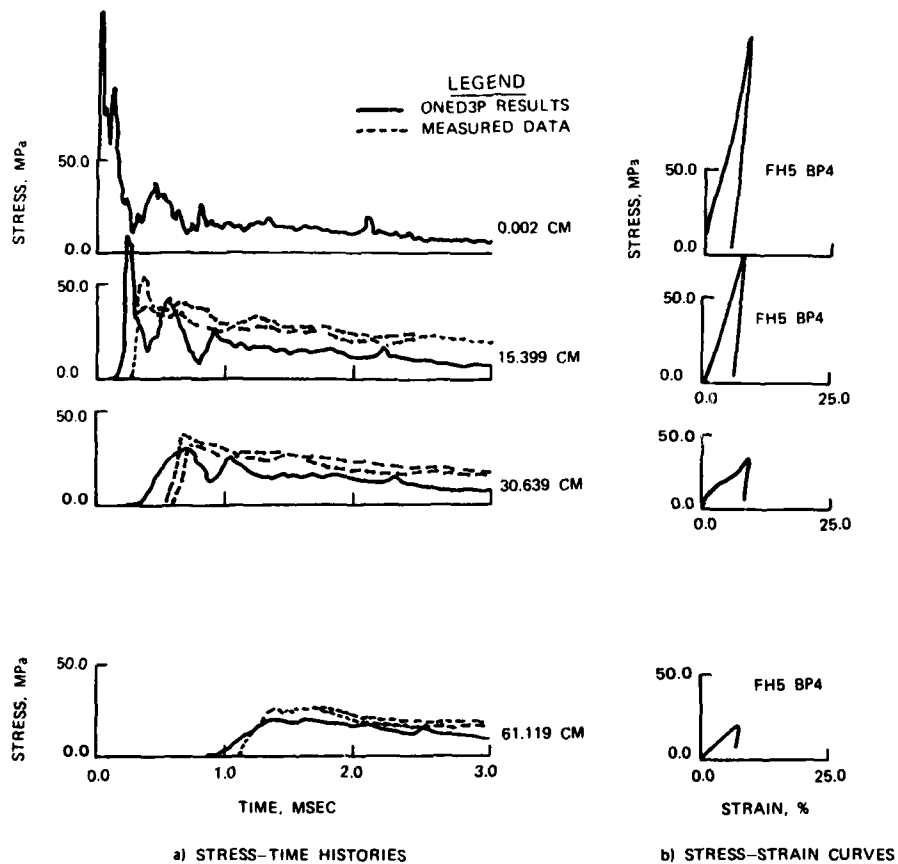


Figure 40. A comparison of ONED3P results with FH5 field test measurements

stress-strain response approaches the "static" response. Initial stress wave velocities predicted by quasi-static laboratory data would have been much smaller than those measured in the field.

63. Note also that the wave speeds calculated for the FH5 field test simulation are greater than those for the FH4 simulations. Since the only difference between the two calculations is the rate of loading (40 microseconds to 21.3 MPa for FH4 and 30 microseconds to 127.7 MPa for FH5), the results demonstrate conclusively that the rate of loading must influence wave speeds by causing a stiffer dynamic stress-strain in the material.

## PART VI: SUMMARY

64. Recent experimental data on soils showing that the application of loads with submillisecond rise times results in significant rate-dependent compressibility behavior have prompted the need for a one-dimensional plane wave propagation code for layered rate-dependent hysteretic or visco-compacting materials. Accordingly an explicit one-dimensional finite element code named ONED3P has been developed which incorporates a three-parameter (spring and dashpot) mechanical model consisting of a linear spring and dashpot in series coupled in parallel with a piecewise linear hysteretic or compacting spring.

65. ONED3P allows for the analysis of a column of multilayered visco-compacting soils loaded by a digitized surface pressure-time history. Any set of consistent units may be used with this code and results may be obtained in the form of stress-, strain-, acceleration-, velocity-, and/or displacement-time histories as well as stress-strain curves using standard Calcomp software.

66. Several demonstration problems were calculated using ONED3P to evaluate its features and capabilities against (a) available analytical solutions for viscous and nonviscous problems, (b) other code solutions, and (c) measurements from field experiments that evince rate-dependent soil behavior. In general, results were extremely good.

67. Special attention was given to the effects of loading rate or frequency on wave speeds in viscous media and to methods of deriving the ONED3P model parameters from laboratory material property test data. Program listings and a user's guide for ONED3P are included in Appendix A.

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9. J. A. Morrison; "Wave Propagation in Rods of Voight Material and Viscoelastic Materials with Three-Parameter Models"; Q. Applied Mathematics; Volume 14; 1956.
10. J. E. Windham; "Stress Transmission During Foam HEST Tests of Sand-Covered Box Structures: Analyses Using a One-Dimensional Plane Wave Code"; Informal report to DNA, October 1980; U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.

## APPENDIX A: WES USER GUIDELINES AND PROGRAM/DRIVER LISTINGS

### ONED3P

1. ONED3P is set up for running on the WES GE 635 computer system CARDIN subsystem\* and requires a free-field data file in the form of a permanent file for execution, whose name should appear on line number 8020 of ONED3P:

8020\$:PRMFL:39,Q,L,ROSDJOC/file name

If there is more than one input quantity on a line of the data file those quantities should be separated by commas or blanks. Any line numbers used to generate the file must be stripped before ONED3P can be executed. Hollerith-type information must be enclosed in double quotes. The format of a typical data file follows.

#### Section 1: General Information

Line	"Problem Title"
Line	Boundary condition

The boundary condition is an integer which selects the type of bottom boundary condition and may have the following values:

1 = free    2 = fixed    3 = transmitting

#### Section 2: Material Properties

Line	"Title Describing Materials"
Line	Number of layers in the soil column

---

\* Execution time for any calculation on the GE 635 may be figured by assuming that ONED3P needs 0.00007 hundredths of an hour of CP time/element/time step.

Line	Layer number, number of elements in layer, layer density, layer height
Line	$n$ , $M_2$ for that layer
Line	Number of stress and strain pairs (of $\sigma$ - $\epsilon$ coordinates) defining the piecewise linear segments of the $M_1$ load curve including the origin number of stress and strain pairs defining the $M_1$ unloading curve
Line(s)	Values of stress-strain pairs defining $M_1$ load curve in sequential order beginning at the origin
Line(s)	Values of stress-strain pairs defining $M_1$ unloading curve beginning at user-chosen point on the load curve

The last five (or more) lines are repeated until all of the layers are defined beginning with layer 1 at the top of the soil column.

### Section 3: Surface Loading Function

Line	"Title"
Line	Number of stress-time data pairs
Line(s)	Stress-time data pairs beginning at the origin

### Section 4: Execution and Print/Plot Parameters

Line	Problem time at which calculation will stop, time increment
Line	Print interval,* plot interval,* number of locations in the column for which plot data are to be saved**
Line	Numbers of the elements or nodes in the column for which plot data are being saved†
Line	Total number of plots requested
Line	Type of plot, element or node number

- 
- \* Print/plot interval takes an integer; 1 means save data at every time step for print/plot; 4 means save data every fourth time step, etc.
  - \*\* The dimension of the FPLOT vector in ONED3P (line 175) limits the amount of plot data which can be saved. That dimension must be greater than 500 plus six times the number of time points to be saved times the number of elements being saved.
  - † For each number listed results are saved for both the element and node having that number.



Line            X value of plot origin in inches, Y value of plot  
                  origin in inches, plot angle in degrees, X axis scale  
                  factor in units/inch of plot, Y axis scale factor,  
                  length of X axis in inches, length of Y axis

The last two lines of data are repeated sequentially until all requested plots have been described. These lines require the following explanation. First of all the plot type is an integer with the following values:

<u>Value</u>	<u>Type of Plot</u>
1	Stress-time history
2	Strain-time history
3	Stress-strain curve
4	Acceleration-time history
5	Velocity-time history
6	Displacement-time history

Next the origin of each plot refers to its position on a standard 34-in. Calcomp drum plot where the origin on the drum plot is located near the bottom edge of the paper. The plot angle may be either zero degrees for plots whose X axis is parallel to the bottom edge of the drum roll or 90, 180, or 270 degrees rotated in a counterclockwise direction.

2. As an example of a typical data file for ONED3P, consider Figure A1 which shows the file that was used for one problem in Part IV.

3. A listing of ONED3P follows.

"MORRISON CHECK NO.1"	}	SECTION 1
3		
"5 M COLUMN: M1 M2=100 MPa, FO=2000"	}	SECTION 2
1		
1,50,2000.,5.		
250.,100.		
2,2		
0.,0.,100.,1.		
100.,1.,0.,0.	}	SECTION 3
"APPLIED STRESSES"		
3	}	SECTION 4
0.,0.,10.,1.,10.,10000.		
20.,0.1	}	SECTION 4
10,1,6		
1,10,20,30,40,50		
6		
1,1		
2.,25.,0.,4.,10.,5.,1.		
1,10		
2.,23.,0.,4.,10.,5.,1.		
1,20		
2.,21.,0.,4.,10.,5.,1.		
1,30		
2.,19.,0.,4.,10.,5.,1.		
1,40		
2.,17.,0.,4.,10.,5.,1.		
1,50		
2.,15.,0.,4.,10.,5.,1.		

Figure A1. ONED3P input file for problem 7,  
Part IV

FILE PAGE NO. 1

A5

```

0490      AMAC(I)=0.
0500      IMAC(I)=0.
0510      IM1(I)=0.
0520      12 EMAC(I)=0.
0530      E1=0.
0540      E2=0.
0544      AP1=0.
0546      AP2=0.
0550
0560
0570
0580
0590      COMPUTE LUMPED MASS VALUES AND INPUT LAYER MATERIAL PROPERTIES
0600
0610      READ(39,1020) TITLE
0620      PRINT,TITLE
0630      READ(39,1020) NL
0640      NMAC=1
0650      DTMIN=1.E5
0660
0670
0680      LN= LAYER NO.; NZ= NO. OF ELEMENTS IN LAYER
0690      PD= LAYER DENSITY; H= LAYER THICKNESS
0700
0710
0720
0730
0740      READ(39,1020) LN,NZ,PD,H
0750      IF(LN.NE.1) STOP
0760      READ(39,1020) ETA(LN),AM2(LN)
0770      READ(39,1020) NSL(1),NSU(1)
0780      READ(39,1020) (SL(I,I),EL(I,I),I=1,NSL(1))
0790      READ(39,1020) (SU(I,I),EU(I,I),I=1,NSU(1))
0800      DC= H*FLOAT(NZ)
0810      AMAC= PD*DC/2.
0820      SLOPE= (SU(1,1)-SU(1,2))/(EU(1,1)-EU(1,2))+ AM2(LN)
0830      CP= 30RT*(SLOPE/PD)
0840      DT= DC/CP
0850      IF(DT.LT.DTMIN) DTMIN=DT
0860      C(I)= 0.0
0870      IF(NZ.EQ.0) GO TO 16
0880      DO 14 I=1,NZ
0890      AMAC(I)= AMAC(I)+ AMAS
0900      AMAC(I+1)= AMAC(I+1)+ AMAS
0910      C(I+1)= C(I)+DC
0920      C(I)= C(I)+ DC/2.
0930      14 NLAY(I)= LN
0940      NMAC= I+1
0950      NEL= NZ
0960      16 IF(NL.EQ.1) GO TO 21
0970      DO 20 J=2,NL
0980      READ(39,1020) LN,NZ,PD,H
0990      IF(LN.NE.J) STOP "LAYER INFO OUT OF ORDER"
1000      READ(39,1020) ETA(LN),AM2(LN)
1010      READ(39,1020) NSL(LN),NSU(LN)
1020      READ(39,1020) (SL(LN,K),EL(LN,K),K=1,NSL(LN))
1030      READ(39,1020) (SU(LN,K),EU(LN,K),K=1,NSU(LN))
1040      DC= H*FLOAT(NZ)
1050      AMAC= PD*DC/2.
1060      SLOPE= (SU(LN,1)-SU(LN,2))/(EU(LN,1)-EU(LN,2))+ AM2(LN)

```

```

1100 CP= SQRT(CLOPE/PO)
1110 DT= DC/CP
1120 IF (DT.LT.DTMIN) DTMIN=DT
1130 DO 18 I=1,N2
1140 AMASS(NMASS)= AMASS(NMASS)+ AMAS
1150 AMASS(NMASS+1)= AMASS(NMASS+1)+ AMAS
1160 C(NMASS+1)= 2*(NMASS)+DC
1170 C(NMASS)= 2*(NMASS)+ DC*0.5
1180 NLAY(NMASS)= LN
1190 NMASS= NMASS+1
1200 18 CONTINUE
1210 NEL= NEL+ N2
1220 20 CONTINUE
1230 21 IF (NBTYP.EQ.3) AMASS(NMASS)= AMASS(NMASS)+ AMAS
1240 CPINIT=CP
1250 CPLAST= CP
1300 PRINT 1030, (I,2*I,AMASS(I),I=1,NMASS)
1320 DUM1(1)=0.
1330 DUM2(1)=0.
1340 NPLT=2
1350C
1360C INPUT SURFACE STRESS TIME HISTORY
1370C
1380 THYME= 0.0
1390 F(1)= ISTPS*(THYME,1)
1400C
1410C ESTABLISH A TABLE OF MODULI FOR THE M1 FUNCTION
1420C
1430 DO 24 I=1,NL
1440 DO 24 N=2,NSL(I)
1450 SLP= (CL(I,N)-CL(I,N-1))/(EL(I,N)-EL(I,N-1))
1460 IF (SLP.LT.0.) STOP " NEGATIVE LOADING MODULUS"
1470 24 CL(I,N-1)= SLP
1480 NM1= NEL(I)-1
1490 DO 26 N=2,NMU(I)
1500 SLP= (SU(I,N-1)-SU(I,N))/(EU(I,N-1)-EU(I,N))
1510 EU(I,N-1)= SU(I,N)
1520 IF (SLP.LT.0.) STOP " NEGATIVE UNLOADING MODULUS"
1530 26 SU(I,N-1)= SLP
1540 NM1= NMU(I)-1
1550 29 CONTINUE
1560 DTMIN= 0.9*DTMIN
1570 CPINIT= SQRT(CL(NL,1)/PO)
1580C
1590C INCREMENT TIME
1600C
1610 PRINT,
1620 PRINT," *****"
1630 PRINT," MAXIMUM TIME INCREMENT SHOULD BE",DTMIN
1640 PRINT," *****"
1650 PRINT,
1660C READ MAXIMUM TIME AND TIME INCREMENT
1670 READ(39,1020) TMAX,DT
1680 DO 2000 I=1,NEL
1690 2000 DTG(I)= DT

```

```

1770      NTIME=1
1780      READ PRINT AND PLOT INTERVALS AND NO. OF PTS. TO BE SAVED
1790      READ(24,1020) NPRINT,NPLTI,KEEP
1792      IF(KEEP.EQ.0) GO TO 4000
1794      READ(24,1020) (NKEEP(I),I=1,KEEP)
1800      NPTC= IMAX*(DT*FLOAT(NPLTI))+1
1802      NICE= NPTC*5*KEEP+500
1804      IF(NICE.GT.25000) (TOP " PLOT FILE TOO BIG"
1810 4000 NPLTI=1
1820      PRINT," NO. OF MASS POINTS=" ,NMASS
1830      PRINT," NO. OF ELEMENTS " ,NEL
1835      NMID= NEL/2+1
1840      GO CONTINUE
1850      DO 34 I=1,NEL
1870      34 LIGN(I)=0
1880      IF(NPLTI.GT.100) GO TO 125
1890
1900      COMPUTE NEW ACCELERATION
1910
1920      DO 60 I=1,NMASS
1930      LN=NLAI(I)
1940      IF(1,NEL,NMASS) GO TO 48
1950      GO TO (48,40,45),NETYPE
1960
1970      FIXED BOTTOM BOUNDARY
1980
1990      40 ACC(I,I)= 0.
2000      GO TO 60
2010
2020      TRANSMITTING BOTTOM BOUNDARY
2030
2035      45 E1= EPS(2,NEL)
2038      IF(ABS(E1).GT.TDEP) GO TO 46
2039      E1=E2
2042      FACT= 1.0
2043      GO TO 47
2044      46 IF(IFLGB.EQ.1) GO TO 47
2045      DAP2= (.16*(2,NEL)+.16*(3,NEL))*0.5*(EPS(2,NEL)-EPS(3,NEL))
2046      IF(EPS(2,NEL).LT.EPS(3,NEL)) IFLGB=1
2047      AP2= AP2+DAP2
2048      FACT= AP2/AP1
2050      47 DV= (DPT*(XMI*FACT/PO)+E1-E2)
2050      E2= E1
2050      GO TO 60
2060
2070      FREE BOTTOM BOUNDARY
2080
2090      48 ACC(I,I)= F(I)*AMASS(I)
2100      GO CONTINUE
2110      IF(MOD(NTIME,NPRINT),NEL.0) GO TO 101
2120      PRINT," TIME=" ,THYME
2130      PRINT,(IG(1,I),EPS(1,I),SIG(1,NMID),EPS(1,NMID),SIG(1,NEL),
2131      (EPS(1,NEL)
2140      101 IF(MOD(NTIME,NPLTI),NEL.0) GO TO 103
2145      IF(KEEP.EQ.0) GO TO 103

```

```

2250 TIME=NPLT* THYME
2252 DO 102 I=1*KEEP
2254 N1= (I-1)*NPT* NPLT
2256 N2= (I-1)*KEEP*NPT* NPLT
2258 N3= (I-1)*2*KEEP*NPT* NPLT
2260 N4= (I-1)*3*KEEP*NPT* NPLT
2262 N5= (I-1)*4*KEEP*NPT* NPLT
2264 I= (KEEP*I)
2266 FFLDT(N1)=IIG*1.*K
2268 FFLDT(N2)=EP*1.*K
2270 FFLDT(N3)=ACC*1.*K
2272 FFLDT(N4)=VEL*1.*K
2274 102 FFLDT(N5)=DIEP*1.*K
2280 NPLT= NPLT+1
2282 103 NTIME= NTIME+1
2284 DO 49 I=1*NMAT
2286 49 F(I)= 0.
2288 THYME= THYME+DT
2290
2292 INTEGRATE ACCELERATION
2294
2296 DO 56 I=1*NMAT
2298 VEL*1,I= VEL*2,I+ ACC*1,I*DT
2300 DIEP*1,I= DIEP*2,I+ VEL*1,I*DT
2302 56 CONTINUE
2304 IF(NBTYPE.EQ.3) VEL*1,NMAT= VEL*2,NMAT+DV
2306
2308 COMPUTE STRAINS AND STRESSES
2310
2312 DO 100 I=1,NEL
2314 NCONT=0
2316 I1=1
2318 LNE= NLAY*I
2320 NHTAH= ETH*LNE
2322 DEPS=DT*(VEL*1,I+VEL*1,I+1)/2*(I+2*(I+1))
2324 IF(ABS(DEPS).GT.TDEPS) GO TO 58
2326 IF(ABS(EP*2,I).GT.TDEPS) GO TO 58
2328 VEL*1,I= VEL*2,I
2330 IF(I.EQ.NEL) VEL*1,I+1= VEL*2,I+1
2332 DEPS=0.
2334 58 EP*1,I= EP*2,I+DEPS
2336 IF(EP*1,I.LT.EMAX(I)) LSIGN(I)=1
2338 EF= EP*1,I
2340 ES= EP*2,I
2342 IF(EP*1,I.GT.EMAX(I).AND.ES.LT.EMAX(I)) LSIGN(I)=1
2344 ELM1= EP*2,I
2346 I2= IIG*2,I
2348 I3M1= IIG*3,I
2350 DTA=0.
2352 DTH= DT
2354 EI= EF
2356 IF(LSIGN(I).EQ.0) GO TO 62
2358 IF(EP*1,I.GT.E3) GO TO 400
2360
2362 UNLOADING

```

```

26300
2635  NMAX= N00*LN*-1
2638  IF*ES*LT*ESM1* GO TO 300
2640  IF*EI*LT*EMAX*I* GO TO 295
2650  ENE*I*1* = EMAX*I*
2660  DO 285 J=1,NMAX
2670  285 IF*CMAX*I*.GT*EU*LN*J* GO TO 287
2680  287 DUMMY= CMAX*I*
2685  IFLAG*I*=J
2690  JMAX= NMAX-J+2
2700  DO 290 NNE= 2,JMAX
2705  DIV= 10*LN*J*
2708  IF*AM2*LN*.LT*1.E6* DIV= DIV+ AM2*LN*
2710  ENE*I*NNE* = ENE*I*NNE-1*-(DUMMY-EU*LN*J*)/DIV
2720  DUMMY= EU*LN*J*
2730  J=J+1
2760  295 ESM1= ES
2770  DSM1= ES
2780  300 SM1= AM1*II*ES*EF*EI*NMAX*I*
2783  IF*ABS*EF-ES*.LT*1.E-10* GO TO 310
2785  DTN= DTN*(EI-ES)/(EF-ES)
2790  310 IF*IP1*(SM1*AM2*LN*)*AYTAH*SS*ESM1*ES*EI*DTN*DT0*I*)*DSM1*
2795  IF*1.E0.NEL* AP1= AP1+(SM1*I*)+DSM1*0.5)*(EI-ES)
2800  SM1*I* = SM1*I* + DSM1
2810  DTA= DTA+ DTN
2811  DT0*I* = DTN
2820  IF*ABS*(DT-DTA)/DT*.LT*1.E-6* GO TO 70
2821  DTA= DT-DTA
2822  DSM1= IF
2823  SS= IF
2830  ESM1= EI
2840  EI= EF
2845  IF*AM2*LN*.GT*1.E8* AYTAH=0.
2850  EI= EF
2851  NCNT= NCNT+1
2852  IF*NCNT*GT*20* PRINT," STOP IN UNLOADING--ELEMENT",I," THYME",
2853  *THYME
2854  IF*NCNT*GT*20* GO TO 125
2860  GO TO 300
29800
29900  PELOADING
30000
3010  400 CONTINUE
3020  NMAX= N00*LN*-1
3030  IF*ES*GT*ESM1* GO TO 410
3040  ESM1= ES
3050  DSM1= ES
3060  410 SM1= AM1*II*ES*EF*EI*NMAX*2*
3063  IF*ABS*(EF-ES).LT*1.E-10* GO TO 415
3065  DTN= DTN*(EI-ES)/(EF-ES)
3070  415 IF*IP1*(SM1*AM2*LN*)*AYTAH*SS*ESM1*ES*EI*DTN*DT0*I*)*DSM1*
3075  IF*1.E0.NEL* AP1= AP1+(SM1*I*)+DSM1*0.5)*(EI-ES)
3350  SM1*I* = SM1*I* + DSM1
3360  DTA= DTA+ DTN
3361  DT0*I* = DTN

```



```

3370 IF (ABS(DT-DTA)/DT).LT.1.E-6) GO TO 420
3371 DTN= DT-DTA
3372 DSM1= EF
3373 EI= EF
3374 DSM1= EI
3375 EI= EI
3376 IF (AM2/LN).GT.1.E8) AYTAN= 0.
3377 EI= EF
3378 NCNT= NCNT+1
3379 IF (NCNT.GT.20) PRINT," STOP IN RELOADING--ELEMENT",I," THYME",THYME
3380 IF (NCNT.GT.20) GO TO 125
3381 GO TO 410
3382 420 IF (EF.LT.EMAX(I)) GO TO 70
3383 EMAX(I)= EF
3384 DSM1(I)= EF
3385 GO TO 70
3386
3387 VIRGIN LOADING
3388
3389 62 CONTINUE
3390 NMAX= NEL/LN-1
3391 63 DSM1= AM1/II*ES*EF*EI*NMAX/3)
3392 IF (ABS(EF-ES).LT.1.E-10) GO TO 64
3393 DTN= DTN+(EI-ES)/(EF-ES)
3394 64 IF (IPI*(SM1*AM2/LN)*AYTAN*DSM1/SS*ESM1*ES*EI*DTN*DTO(I)*DSM1)
3395 IF (1.E0/NEL) AR1= AR1+(SM1(I)+DSM1*0.5)*(EI-ES)
3396 DSM1(I)= DSM1(I)+ DSM1
3397 DTA= DTA+ DTN
3398 DTO(I)= DTN
3399 IF (ABS(DT-DTA)/DT).LT.1.E-6) GO TO 65
3400 DTN= DT-DTA
3401 DSM1= EF
3402 EI= EF
3403 DSM1= EI
3404 EI= EI
3405 IF (AM2/LN).GT.1.E8) AYTAN= 0.
3406 EI= EF
3407 NCNT= NCNT+1
3408 IF (NCNT.GT.20) PRINT," STOP IN VIRGIN LOADING--ELEMENT",I,
3409 " THYME",THYME
3410 IF (NCNT.GT.20) GO TO 125
3411 GO TO 63
3412 65 DSM1(I)= EF
3413 EMAX(I)= EF
3414 70 SIG(2,I)= SS
3415 SIG(3,I)= DSM1
3416 EPS(2,I)= ES
3417 EPS(3,I)= ESM1
3418 SIG(1,I)= EF
3419 F(I)= F(I)-SIG(1,I)
3420 F(I+1)= F(I+1) + SIG(1,I)
3421 100 CONTINUE
3422
3423 DO 110 I=1,NMAX
3424 ACC(2,I)= ACC(1,I)

```

```

4050      VEL(2,I)=VEL(1,I)
4060      DIER(2,I)=DIER(1,I)
4070 110 CONTINUE
4080      DO 115 I=1,NEL
4090          IIG(3,I)=IIG(2,I)
4100          IIG(2,I)=IIG(1,I)
4110          EPI(3,I)=EPI(2,I)
4120          EPI(2,I)=EPI(1,I)
4130 115 CONTINUE
4135      DTG(I)=DTN
41400
41500      ADD SURFACE STRESS
41600
4170      F(1)=F(1)+DITPE*THYME*NTIME
4180      IF(NPLT.LT.1000.AND.THYME.LT.TMAX) GO TO 30
41900
42000      CALCULATE STRESS POINT LOCATION
42100
4220      DO 122 J=2,NMASS
4230 122  CS(J)=(C(J)+2*(J-1))*0.5
4240      NTM1=NTIME-1
42500
42600      PLOTTER OUTPUT
42700
4280 125 CALL PLOTS(0,0,3)
4290      READ(39,1020) NPLOTS
4295      PRINT," NO. PLOTS=",NPLOTS
4300      IF(NPLOTS.EQ.0) GO TO 160
4310      NPLT=NPLT+1
4320      DO 150 M=1,NPLOTS
4330 130 READ(39,1020) N1,N2
4340      IF(N1.EQ.0) GO TO 160
4350      DO 131 I=1,KEEP
4360      IF(NKEEP(I).NE.N2) GO TO 131
4370      N2=I
4380 131 CONTINUE
4390      GO TO (132,135,138,141,144,147),N1
4500 132 PRINT," STRESS-TIME",N2
4510      DO 133 I=1,NPLT
4520      N=(N2-1)*NPTS+I
4530 133 DUM1(I)=FPLOT(N)
4540      CALL PLOTT(TIME,I),DUM1(I),NPLT,1,2,PTITL)
4550      GO TO 150
4560 135 PRINT," STRAIN-TIME",N2
4570      DO 136 I=1,NPLT
4580      N=(N2-1+KEEP)*NPTS+I
4590 136 DUM1(I)=FPLOT(N)*100.
4610      CALL PLOTT(TIME,I),DUM1(I),NPLT,1,3,PTITL)
4620      GO TO 150
4630 138 PRINT," STRESS-STRAIN",N2
4640      DO 139 I=1,NPLT
4650      N=(N2-1)*NPTS+I
4660      NN=(N2-1+KEEP)*NPTS+I
4670      DUM2(I)=FPLOT(N)
4680 139 DUM1(I)=FPLOT(NN)*100.

```

```

4700      CALL PLOTT(DUM1*1,DUM2*1,NFLT,3*2,PTITLE)
4710      GO TO 150
4720 141 PRINT," ACC-TIME",N2
4730      DO 142 I=1,NFLT
4740      N=(N2-1+3*KEEP+NFTE+I
4750 142 DUM1*1= FFLOT(N*4.806E-6
4760      CALL PLOTT(TIME*1,DUM1*1,NFLT,1*4,PTITLE)
4770      GO TO 150
4780 144 PRINT," VEL-TIME",N2
4790      DO 145 I=1,NFLT
4800      N=(N2-1+3*KEEP+NFTE+I
4810 145 DUM1*1= FFLOT(N*100.
4820      CALL PLOTT(TIME*1,DUM1*1,NFLT,1*5,PTITLE)
4830      GO TO 150
4840 147 PRINT," DIFP-TIME",N2
4850      DO 148 I=1,NFLT
4860      N=(N2-1+4*KEEP+NFTE+I
4870 148 DUM1*1= FFLOT(N*100.
4880      CALL PLOTT(TIME*1,DUM1*1,NFLT,1*6,PTITLE)
4890 150 CONTINUE
4900 180 REMIND 39
4910      CALL DETACH 39...
4920      PRINT," DONE-DONE-DONE"
4930      CALL PLOT(0.0,0.999)
4940      REMIND 3
4950      STOP
4960 1000 FORMAT(I24,"")
4970 1020 FORMAT(V)
4980 1030 FORMAT(I10,2F12.5)
4990 1040 FORMAT," LAYER",I5," VISCIDITY=",F15.3," M2=",F15.2)
5000 1050 FORMAT(70,"MODE",6X,"D1G",9X,"EPS",9X,"ACC")
5010 1060 FORMAT(I10,4F12.5,F12.7)
5020      END
5030
5040      FUNCTION COTR(XT,N)
5050      CHARACTER TITLE*85
5060      DIMENSION Y(600),Y1(600)
5070      IF(XT.LT.1) GO TO 10
5080      READ(39,2000) TITLE
5090      READ(39,2000) NFTE
5100      READ(39,2000) (Y(I)*X(I),I=1,NFTE)
5110      IF=2
5120 10 CONTINUE
5130      DO 20 I=1,NFTE
5140 20 IF(XT.LT.1.0001*X(I)) GO TO 30
5150      Y1(I)= Y(NFTE)
5160      RETURN
5170 30 COTR= Y(I-1)+(Y(I)-Y(I-1))*XT-X(I-1)*(X(I)-X(I-1))
5180      IF= 1
5190      RETURN
5200 200 FORMAT(V)
5210 210 FORMAT(1F4E12.7)
5220      END
5230

```



```

7180 D2= DIM1+R
7190 D3= (EIP1-EI)+L
7200 D4= (EI-EIM1)+E
7210 IIP1= D1-D2+D3-D4
7220 DIM1= C1+(EIP1-EI)
7230 RETURN
7240 END
7250
7260
7270 SUBROUTINE PLOTT(X,Y,N,ILXT,ILYT,TITLE)
7280 CHARACTER *1 AHD,LABL*15
7290 DIMENSION IX(1),Y(1),LABL(7),NC(7),NDP(7),NDX(4),NDY(4),
7300 DIMENSION INCX(4),INCY(4),ISWX(4),ISVY(4),IIX(4),IIY(4),
7310 DIMENSION AT(4),XT(4),YT(4)
7320 DATA INCX /1,-1,1,1/,INCY /1,-1,-1,1/,ISWX /0,1,1,0/,
7330 ISVY /0,0,1,1/,IIX /1,-1,-1,1/,IIY /1,1,-1,-1/
7340 DATA NDX /0,-1,-1,0/,NDY /0,0,-1,-1/
7350 DATA AT /0,90,0,270/,XT /0,-.9,-1.2,.9/,YT /0,.9,-.9,-.7/
7360 DATA NC /11,12,10,9,13,9,10/,NDP /1,2,1,1,1,1,2/
7370 LABL(1)= "TIME - MSEC"
7380 LABL(2)= "STRESS - MPA"
7390 LABL(3)= "TSTRN - %"
7400 LABL(4)= "ACC - G S"
7410 LABL(5)= "VEL - CM/MSEC"
7420 LABL(6)= "DISP - CM"
7430 LABL(7)= "DEPTH - CM"
7440 ILX= ILXT
7450 ILY= ILYT
7460 READ(39,200) X0,Y0,ANGLE,IFX,IFY,XL,YL
7470 DO= IFX
7480 DY= IFY
7490 IA= (ANGLE+1)/90+1
7500 THETA= ANGLE*3.14159265/180.
7510 C= COS(THETA)
7520 S= SIN(THETA)
7530 CALL PLOT(X0,Y0,-3)
7540 IF IA.NE.2.AND. IA.NE.4) GO TO 2
7550 IDUM= ILX
7560 ILX=ILY
7570 ILY=IDUM
7580 XDUM= DX
7590 DX= DY
7600 DY= XDUM
7610 XDUM= XL
7620 XL=YL
7630 YL=XDUM
7640 2 NDX= NC+ILX+INCX+IA
7650 NDY= NC+ILY+INCY+IA
7660 ISWX= DX+ISWX+IA+XL
7670 ISVY= DY+ISVY+IA+YL
7680 IIX= DX+IIX+IA
7690 IIY= DY+IIY+IA
7700 ISWX= XL+NDX+IA
7710 ISVY= YL+NDY+IA
7720 IS= XT+IA+XL

```

DNEDEBP

CONT

10: 5:31 08 26 81

FILE PAGE NO. 12

```
7774      VC=AT*IA*VL
7776      CALL SYMBOL(12,VC,107,TITLE,AT*IA*40)
7780      CALL R0113(000,0,LABEL,ILN*NOV,108,VL,NDR,ILN*0,DV*IL*1,1,1,0)
7790      CALL R0113(0,0,ZVY,LABEL,ILY*NOV,108,VL,NDR,ILY*1,DV*DY*1,1,1,0)
7795      CALL PLOT(0,0,3)
7800      5 DO 10 I=1,N
7810      H= (X(I)*C1-IFX-Y(I)*C1-IFV)
7820      V= (X(I)*C1-IFX+Y(I)*C1-IFV)
7830      CALL PLOT(H,V*2)
7840      10 CONTINUE
7850      CALL PLOT(-30,-Y0,-3)
7870      RETURN
7875      200 FORMAT(V)
7880      END
79903:OIE:JGLIT
80003:EXECUTE
80103:LIMIT:30.700
80203:PMFL:39.0*L*P01DJ00*FXDDATA
80303:TAPET:03.020....*CPL
80503:INCODE: IEMF
80603:ENDJOB
```

ONED3PD

4. ONED3PD is the driver program mentioned in Part V, which can be used in a trial-and-error process to establish values of the viscous parameter  $\eta$  and the spring  $M_2$ . This code is presently set up to run on the WES DPS/1 time-sharing computer system and makes use of a data file which is very similar to that required for ONED3P; however, because the time-sharing system is interactive, many of the input quantities required for running ONED3PD are input from a teletype keyboard by the user.

5. Referring to the data file shown on Figure A1 for ONED3P batch runs, all that is required for a ONED3PD data file is the problem title from Section 1, the static stress-strain curve ( $M_1$  function) from Section 2, and the forcing function from Section 3. All other input information is asked for by ONED3PD at the time of execution through the process of interactive questions and answers. A typical ONED3PD data file follows:

```

" FH4 04.6"
3 3
0 0 26.1076 35.2 .118
35.2 .118 0. .108
" LOADING FUNCTION NFA-MEC"
29
0. 0. 0.203 0.041 0.626 0.086 0.834 0.108
1.468 0.130 2.934 0.152 3.127 0.174 6.045 0.197
13.543 0.219 17.301 0.241 18.343 0.263 20.641 0.286
29.547 0.308 35.587 0.330 36.352 0.352 42.931 0.374
42.516 0.387 43.134 0.419 40.259 0.441 37.913 0.463
27.251 0.446 23.129 0.508 20.254 0.530 14.383 0.552
12.503 0.574 8.752 0.587 9.124 0.619 5.837 0.641
3.543 0.663

```

6. A listing of ONED3PD follows.

0010 \*DNEEDP\*19901441 PLOT1-P

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DNEEDP--DRIVER FOR DNEEDP

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ONE-DIMENSIONAL BEHAVIOR OF A POINT OF VISCOELASTIC MAT.  
THE CONSTITUTIVE BEHAVIOR MAY BE REPRESENTED BY A MECHANICAL  
MODEL CONSISTING OF A PARALLEL SPRING AND MAXWELL ELEMENT

UNITS ARE ANY SET OF CONSISTENT UNITS; FOR INSTANCE,  
MEGAPASCALS, MSEC, METERS, KG

CHARACTER\*1 AND TITLE\*65\*FILE\*24\*PTITL\*40  
COMMON SL(50),SU(50),EL(50),EU(50),ENE(10),IFLAG,NEL,NEU  
DIMENSION TIME(500),DUM1(500),DUM2(500),EPS(3),SIG(3)  
DIMENSION FELOT(10000)  
CALL FPARAM(1,80)  
CALL FNDFT(67,1,1,0)

PRINT,  
5 PRINT," INPUT FILE NAME"  
READ,FILE  
ENCODE,FILE,1000  
CALL ATTACH(3,FILE,3,,)

READ(3,1020) PTITL

PRINT,  
PRINT,  
PRINT,PTITL  
PRINT,

DO 10 I=1,3  
SIG(I)=0.  
EPS(I)=0.

10 CONTINUE

SMAX=0.  
SM1= 0.  
EMAX=0.  
S1= 0.  
S2= 0.

PRINT," ETA\*ME"  
READ,ETA,AME  
READ(3,1020) NEL,NEU  
READ(3,1020) (SL(I),EL(I),I=1,NEL)  
READ(3,1020) (SU(I),EU(I),I=1,NEU)  
DUM1(1)=0.  
DUM2(1)=0.  
NFLT=0

INPUT SURFACE STRESS TIME HISTORY

THYME= 0.0  
SIG(1)= 1000\*THYME,1)

ESTABLISH A TABLE OF MODULI FOR THE M1 FUNCTION



```

14200 DO 24 N=2,NIL
14300   CLP= (CL(N)-EL(N-1))*EL(N)-EL(N-1)
14400   IF (CLP.LT.0.) STOP "NEGATIVE LOADING MODULUS"
14500   24 CL(N-1)= CLP
14600   NM1= NIL -1
14700   DO 26 M=2,NIL
14800     CLP= (CU(N-1)-EU(N-1))*EU(N-1)-EU(N-1)
14900     EU(N-1)= CU(N)
15000     IF (CLP.LT.0.) STOP "NEGATIVE UNLOADING MODULUS"
15100     26 CU(N-1)= CLP
15200     NM1= NM1 -1
15300     PRINT," MAXIMUM TIME"
15400     READ, TMAX
15500     PRINT," TIME INCREMENT"
15600     READ, DT
15700     DTG= DT
15800     NTIME=1
15900     PRINT," PRINT INTERVAL AND PLOT INTERVAL"
16000     READ,NPRINT,NPLTI
16100     NPTS= TMAX*(DT*FLOAT(NPLTI))+1
16200     NICE= NPTS*2+500
16300     IF (NICE.GT.10000) STOP "PLOT FILE TOO BIG"
16400     NPLT=1
16500     30 CONTINUE
16600     LSIGN= 0
16700     IF (NPLT.GT.1000) GO TO 125
16800
16900     IF (MOD (NTIME,NPRINT).NE.0) GO TO 101
17000     PRINT,THME,(16*2)*EPC(2)
17100     101 IF (MOD (NTIME,NPLTI).NE.0) GO TO 103
17200     THME= NPLTI
17300     N1= 2*(NPLT-1)+1
17400     N2= N1+1
17500     FPLOT(N1)= SIG(2)
17600     FPLOT(N2)= EPC(2)
17700     NPLT= NPLT+1
17800     103 NTIME= NTIME+1
17900     THME= THME+DT
18000
18100     COMPUTE NEW STRAIN
18200
18300     NONT=0
18400     IF= SIG(1)
18500     RYTAH= ETP
18600     IF (EPC(2).LT.EMAX) LSIGN= 1
18700     ES= EPC(2)
18800     ESM1= EPC(3)
18900     IF (ES.GT.EMAX.AND.ESM1.LT.EMAX) LSIGN=1
19000     IF= SIG(2)
19100     ESM1= SIG(3)
19200     DTG= DT
19300     IF (LSIGN.EQ.0) GO TO 66
19400     IF (E1.GT.ESM1) GO TO 400

```

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3610
3620
3630
3640 NMAX= NTU-1
3650 IF (ES.LT.ESM1.AND.ESM1.LT.EMAX) GO TO 366
3660 ENE=1+EMAX
3670 DO 368 J=1,NMAX
3680 285 IF (EMAX.GT.EU(J)) GO TO 287
3690 287 DUMMY=EMAX
3700 IFLAG=J
3710 JMAX=NMAX-J+2
3720 DO 290 NNE=2,JMAX
3730 DIV=EU(J)
3740 IF (AM2.LT.1.E6) DIV=DIV+AM2
3750 ENE(NNE)=ENE(NNE-1)-(DUMMY-EU(J))/DIV
3760 DUMMY=EU(J)
3770 290 J=J+1
3780 300 NM1=AM1+ESM1+ES,NMAX,1)
3790 510 EF=EIP1(NM1,AM2,AYTAN,SSM1,SS,SF,ESM1+ES,PTN,DTG,DEM1)
3800 SM1=SM1+DEM1
3810 GO TO 70
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UNLOADING

RELOADING

VIRGIN LOADING

AND SURFACE STRESS

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4140      NIMI= NTIME-1
4150
4160      PLOTTER OUTPUT
4170
4180 125 PRINT," DO YOU WANT PLOTTED OUTPUT?"
4190      READ,RHS
4200      IF (RHS.EQ."Y") GO TO 160
4210      NPLT= NPLT+1
4220 130 PRINT," NEXT PLOT-----TYPE CODE"
4230      READ,N1
4240      IF (N1.EQ.0) GO TO 160
4250      GO TO (133,135,137)N1
4260 133 PRINT," STRESS-TIME"
4270      DO 137 I=1,NPLT
4280      N= 1*(I-1)+1
4290 137 DUM1(I)= FPLDT(N)
4300      CALL PLOTT(TIME(I),DUM1(I),NPLT,1,2,PTITLE)
4310      GO TO 130
4320 135 PRINT," STRAIN-TIME"
4330      DO 136 I=1,NPLT
4340      N= 2*(I-1)+2
4350 136 DUM1(I)= FPLDT(N)+100.
4360      CALL PLOTT(TIME(I),DUM1(I),NPLT,1,3,PTITLE)
4370      GO TO 130
4380 137 PRINT," STRESS-STRAIN"
4390      DO 139 I=1,NPLT
4400      N= 2*(I-1)+1
4410      NN= N+1
4420      DUM2(I)= FPLDT(NN)
4430 139 DUM1(I)= FPLDT(NN)+100.
4440      CALL PLOTT(DUM1(I),DUM2(I),NPLT,3,2,PTITLE)
4450      GO TO 130
4460 160 REWIND 39
4470      CALL DETACH(39,,)
4480      PRINT," DONE-DONE-DONE"
4490      PRINT," DO YOU WANT ANOTHER RUN?"
4500      READ, RHS
4510      IF (RHS.EQ."Y") GO TO 5
4520      STOP
4530 1000 FORMAT(24,"1")
4540 1020 FORMAT(Y)
4550 1030 FORMAT(110,2F12.5)
4560 1040 FORMAT(" LAYER",I5," VISCOSITY=",F15.3," M2=",F15.2)
4570 1050 FORMAT(7X,"NODE",6X,"SIG",9X,"EPS",9X,"ACC")
4580 1060 FORMAT(110,4F12.5,F12.7)
4590      END
4600
4610      FUNCTION STRESS(T,N)
4620      CHARACTER TITLE*65
4630      DIMENSION X(600),Y(600)
4640      IF (N.GT.1) GO TO 10
4650      READ(39,200) TITLE
4660      READ(39,200) NPTS
4670      READ(39,200) (Y(I),X(I),I=1,NPTS)

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A22

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7270 D2= (EIP1-EI)*A
7271 D3= (C1-SIM1)*B
7272 D4= (EI-EIM1)*C
7273 EIP1= EI+D2-D3+D4
7274 DEM1= C1*(EIP1-EI)
7275 RETURN
7276 END
7277
7278 SUBROUTINE PLOTT(X,Y,N,ILXT,ILYT,TITLE)
7279 CHARACTER *1 ANS,LABEL*15
7280 DIMENSION X(1),Y(1),LABEL(7),NC(7),NDF(7),NZX(4),NZY(4)
7281 DIMENSION INCX(4),INCY(4),ISVX(4),ISVY(4),IIX(4),IIY(4)
7282 DIMENSION AT(4),XT(4),YT(4)
7283 DATA INCX/-1,-1,1,1/,INCY/1,-1,-1,1/,ISVX/0,1,1,0/,
7284 ISVY/0,0,1,1/,IIX/1,-1,-1,1/,IIY/1,1,-1,-1/
7285 DATA NZX/0,-1,-1,0/,NZY/0,0,-1,-1/
7286 DATA AT/0.,50.,0.,270./,XT/.7,-.9,-1.2,.9/,YT/.9,-.7,-.9,-.7/
7287 DATA NC/11,12,10,9,13,9,10/,NDF/1,1,1,1,1,2/
7288 LABEL(1)= "TIME - msec"
7289 LABEL(2)= "STRESS - MPa"
7290 LABEL(3)= "STRAIN -%"
7291 PRINT," WANT PLOT?"
7292 READ, ANS
7293 IF (ANS.NE."Y") RETURN
7294 ILX= ILXT
7295 ILY= ILYT
7296 PRINT," READ X0,Y0,THETA,SFX,SFY,XAXISL,YAXISL"
7297 READ,X0,Y0,NGLE,SFX,SFY,XL,YL
7298 DX= SFX
7299 DY= SFY
7300 IA= (NGLE+1)/90+1
7301 THETA= NGLE*.14159265/180.
7302 C= COS(THETA)
7303 S=SIN(THETA)
7304 PRINT," DO YOU WANT THE AXES DRAWN?"
7305 READ, ANS
7306 CALL PLOTS("A")
7307 CALL PLOT(X0,Y0,-3)
7308 IF (ANS.NE."Y") GO TO 5
7309 IF (IA.NE.2.AND. IA.NE.4) GO TO 2
7310 IDUM= ILX
7311 ILX=ILY
7312 ILY=IDUM
7313 XDUM= DX
7314 DX= DY
7315 DY= XDUM
7316 XDUM= XL
7317 XL=YL
7318 YL=XDUM
7319 2 DX= NC(ILX)*INCX(IA)
7320 DY= NC(ILY)*INCY(IA)
7321 DX= DX+ISVX(IA)*XL
7322 DY= DY+ISVY(IA)*YL
7323 DX= DX+IIX(IA)

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7760 DY= DY+I1Y(IA)
7770 DX= XL+NZX(IA)
7780 ZY= YL+NZY(IA)
7790 ND= NT(IA)*XL
7800 VD= VT(IA)*YL
7810 CALL SYMBOL(ND,VD,.07,TITL,AT(IA),40)
7820 CALL AXIS13(ZX,.0,LABEL(ILX),NCX,.1,XL,BDF(ILX),0,DV1,DX,.1,.1,0)
7830 CALL AXIS13(0,ZY,LABEL(ILY),NCY,.1,YL,BDF(ILY),1,DV1,DY,.1,.1,0)
7840 5 DO 10 I=1,N
7850 H= (X(I)*C/SFX-Y(I)*S/SFY)
7860 V= (X(I)*S/SFX+Y(I)*C/SFY)
7870 CALL PLOT(H,V,2)
7880 10 CONTINUE
7890 CALL PLOT(-X0,-Y0,-3)
7900 CALL PLOT(0,0,.999)
7910 RETURN
7920 END
```

In accordance with letter from DAEN-RDC, DAEN-ASI dated 22 July 1977, Subject: Facsimile Catalog Cards for Laboratory Technical Publications, a facsimile catalog card in Library of Congress MARC format is reproduced below.

Curtis, John O.

A one-dimensional plane wave propagation code for layered rate-dependent hysteretic materials / by John O. Curtis (Structures Laboratory, U.S. Army Engineer Waterways Experiment Station). -- Vicksburg, Miss. : The Station ; Springfield, Va. ; available from NTIS, 1981.

66, 24 p. : ill. ; 27 cm. -- (Miscellaneous paper / U.S. Army Engineer Waterways Experiment Station ; SL-81-25)

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"September 1981."

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Final report.

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I. United States. Dept. of the Army. Assistant Secretary of the Army (R&D). II. U.S. Army Engineer Waterways

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